Itasdi

Innovative Teaching Approaches in development of Software Designed Instrumentation and its application in real-time systems

Theory of Robotics Systems

Kinematic control of differential drive mobile robot

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Innovative Teaching Approaches in development of Software Designed Instrumentation and its application in real-time systems

Faculty of Technical Sciences

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School of Electrical Engineering
University of Belgrade

Faculty of Physics
Warsaw University of Technology

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Innovative Teaching Approaches in development of Software Designed Instrumentation and its application in real-time systems

Teorija Robotskih Sistema

Kinematska kontrola
mobilnih robota sa diferencijalnim pogonom

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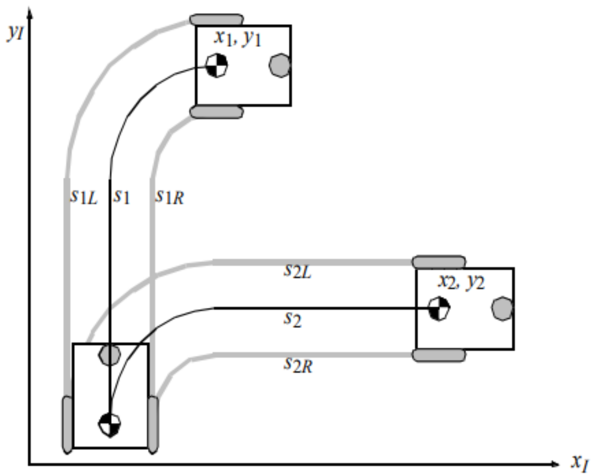
Kinematika mobilnih robota

- Kinematika manipulatora vs kinematika mobilnih robota
  - Predstavljaju odnoso između unutrašnjih i spoljašnjih koordianta
  - Kod mobilnih robota, očitavanja sa enkodera ne mapiraju kretanje u jedinstvenu poziciju u prostoru – Ne postoji direktan način merenja pozicije robota već se pozicija dobija integraljenjem i zavisi pređenog puta.



Introduction to Autonomous Mobile Robots Roland Siegwart and Illah R. Nourbakhsh

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Forward Kinematics – Inverse Kinematikcs

- Forward Kinematics:
  - Vrši transformaciju iz unutrašnjih koordinata u spoljašnje:

    brzina točkova → brzina robota
- Inverse Kinematics:
  - Transformacija iz spoljašnjih koordinata u unutrašnje (potrebna je za implementaciju kontrolera):

    brzina robota → brzina točkova

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Kinematika mobilnih robota

- Upravljamo lokalnim brzinama robota (V i W):
  - Zbog samo konstrukcije robota moguće se robot kreće duž svoje X-ose i okreće oko svoje Z-ose.
- Potrebna nam je informacija o kretanju robota u spoljašnjim koordinatama:
  - Integracijom brzina izraženih u globalnom referentnom sistemu možemo dobiti informaciju o trenutnoj poziciji robota.

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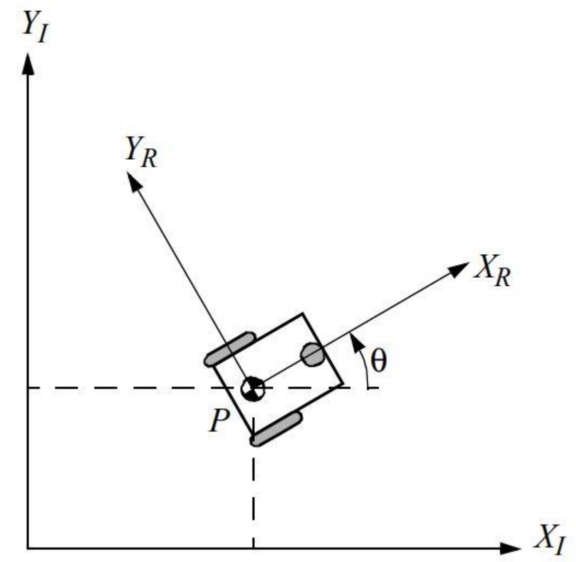
Kinematika mobilnih robota

- $\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$ - pozicija robota u globalnom referentnom sistemu
- Pošto se kretanje robota vrši u ravni XY veza između globalnog i lokalnog koordinatnog sistema može se predstaviti matricom rotacije
- $R = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- $\dot{\xi}_R = R(\theta)\dot{\xi}_I$ odnosno $\dot{\xi}_I = R(\theta)^{-1}\dot{\xi}_R$



Kinematika mobilnih robota – Robot na diferencijalni pogon

- $\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \dot{\varphi}_l, \dot{\varphi}_r)$ – l (polu-rastojanje između točkova), r(prečnik točka)
- Doprinosi svakog točka na pojedinu komponentu brzine:
  - $\dot{X}_{R1} = \frac{1}{2} r\dot{\varphi}_r, \dot{X}_{R2} = \frac{1}{2} r\dot{\varphi}_l$
  - $\dot{Y}_R = 0$
  - $\omega_1 = \frac{r\dot{\varphi}_r}{2l}, \omega_2 = \frac{-r\dot{\varphi}_l}{2l}$
- $\dot{\xi}_I = R(\theta)^{-1} \begin{bmatrix} \frac{r\dot{\varphi}_r}{2} + \frac{r\dot{\varphi}_l}{2} \\ 0 \\ \frac{r\dot{\varphi}_r}{2l} + \frac{-r\dot{\varphi}_l}{2l} \end{bmatrix}$

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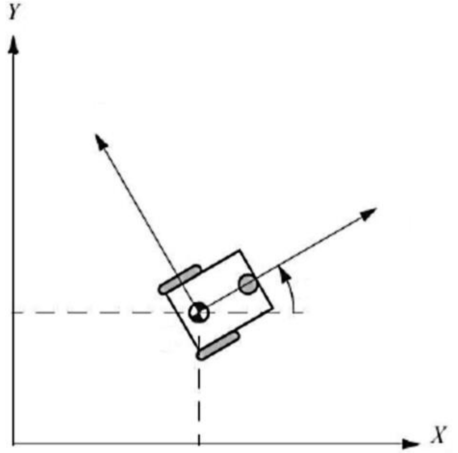
Feedforward kontrola

- Upotreba: Udaljena kontrola robota, predefinisan oblik trajektorije.
- $v = \frac{r\dot{\varphi}_R + r\dot{\varphi}_L}{2}, \omega = \frac{r\dot{\varphi}_R - r\dot{\varphi}_L}{l}$
- Putanja se mora podeliti u jasne segmente: prave linije ili delove kruga.
- Izračunavanje brzina točkova prema zadatoj putanji pre pokretanja programa.
- Nedostaci:
  - Izračunavanje ostvarive trajektorije nekada može da bude težak zadatak.
  - Adaptacija na dinamičke promene u okruženju nije moguća.
  - Rezultujuća trajektorija u krajnjem slučaju nije glatka.



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Upravljanje u zatvorenoj sprezi

$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = K \cdot e = K \cdot {}^{R}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

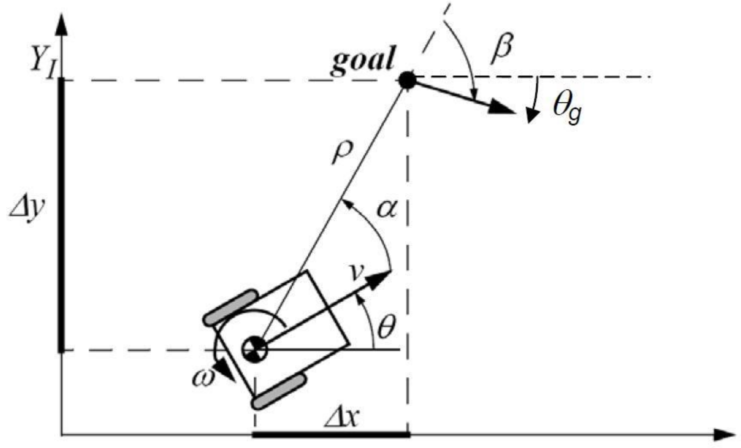
Potrebno je pronaći takvu matricu K,
koja će određenog vremena dovesti grešku na nulu.

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\alpha = -\theta + \text{atan}2(\Delta y, \Delta x)$$

$$\beta = -\theta - \alpha$$

Transformacija u polarne koordinate



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Upravljanje u zatvorenoj sprezi

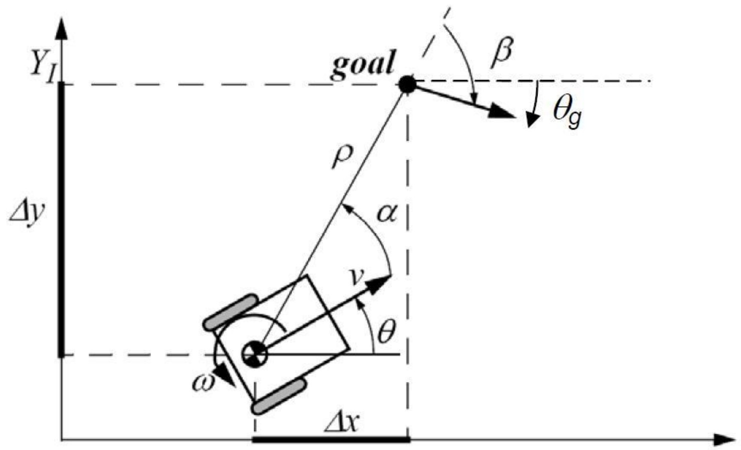
Prelaskom na polarne koordinate translatornu i rotacionu izražavamo:

$$v = k_\rho \rho \qquad \omega = k_\alpha \alpha + k_\beta \beta$$

Gde sistem u zatvorenoj sprezi ima oblik:

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho \rho \cos\alpha \\ k_\rho \sin\alpha - k_\alpha \alpha - k_\beta \beta \\ -k_\rho \sin\alpha \end{bmatrix}$$

$$k_\rho > 0 \;;\quad -k_\beta > 0 \;;\quad k_\alpha - k_\rho > 0$$



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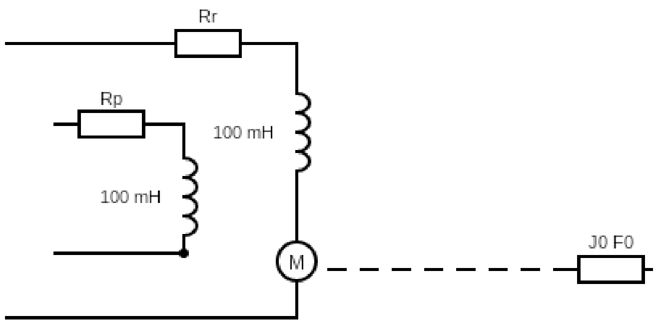
Pogonski mehanizam

- Najčešće se koriste DC motori zbog jednostavnosti kontrole.



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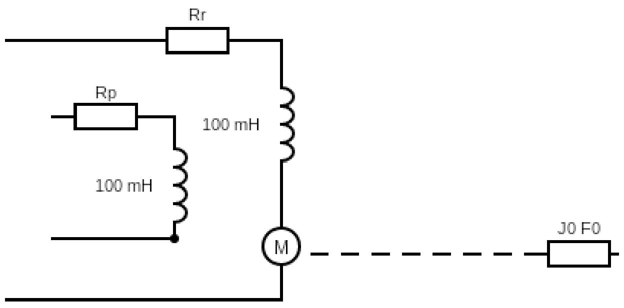
Pogonski mehanizam

- Dovođenjem napona na stator motora uspostavljaju se relacija:

- $R_p i_p(t) + L_p \frac{di_p(t)}{dt} = u_p(t)$
- $I_P = \frac{U_p}{L_p s + R_p} \Rightarrow \Phi(t) \sim i_p(t)$



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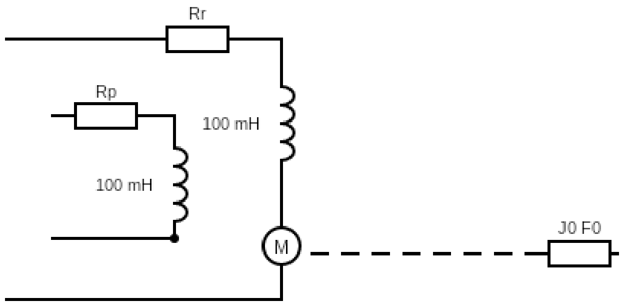
Pogonski mehanizam

- Takođe postoji napon na priključku rotora u_r pri čemu kroz kolo rotora počinje protok struje.
- Proticanje struje u magnetnom kolu

prouzrokovaće stvaranje momenta:



- $M(t) \sim \Phi(\mathrm{t}) i_r(t)$

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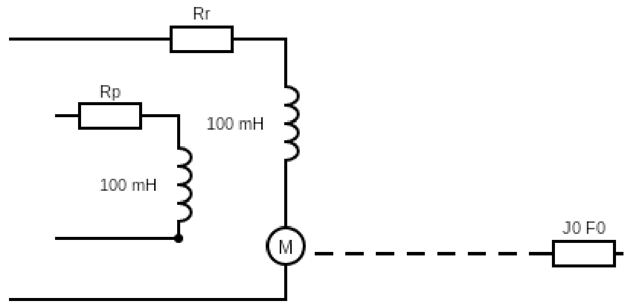
Pogonski mehanizam

- Usled okretanja rotorskog namotaja brzinom $\omega_m(t)$ stvara se elektromagnetska sila:

- $e(t) \sim \Phi(t)\omega_m(t)$



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Pogonski mehanizam

- Upravljanje strujom rotora:
- $i_p(t) = const, \Phi(t) = const$
- $M(t) = K_{em} i_r(t), e(t) = K_{me} \omega_m(t);$ K_{em}-iz električne u momenat

  K_{me}-iz brzine u el.motornu silu

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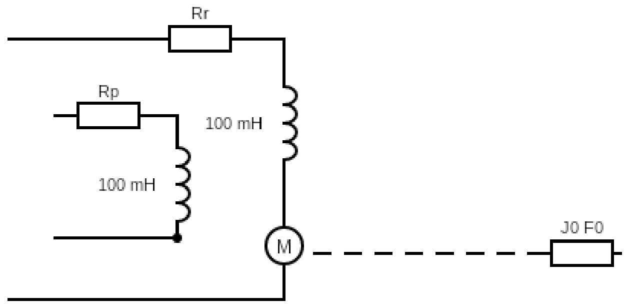
Pogonski mehanizam

- $R_r i_r(t) + L_r \frac{di_r(t)}{dt} + K_{me}\omega_m(t) = u_r(t)$ – Omov zakon za rotorski namotaj
- Mehanička ravnoteža $M(t) = J_e \frac{d\omega_m(t)}{dt} + F_e\omega_m(t)$
- $-K_{em} i_r(t) + J_e \frac{d\omega_m(t)}{dt} + F_e\omega_m(t) = 0$



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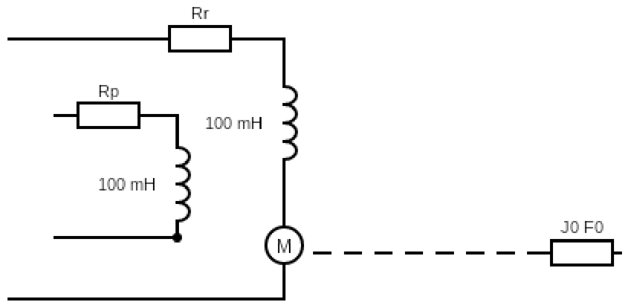
Pogonski mehanizam

- Jednačine napisane nakon Laplasove transformacije:
- $(R_r + L_r s)I_r(t) + K_{me}\omega_m(s) = U_r(s)$
- $-K_{em}I_r(s) + (J_e s + F_e)\omega_m(s) = 0$
- Ulaz napon na rotoru izlaz iz sistema brzina motora:
- $G_m(s) = \frac{\omega_m(s)}{U_r(s)}$
- $G_m(s) = \frac{K_{em}}{(L_r s+R_r)(J_e s+F_e)+K_{em}K_{me}}$



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Pogonski mehanizam

- Sistem u otvorenoj sprezi ne može da odreaguje na promenu opterećenja koje motor „vidi“.
- Možemo uvrstiti kao ulaz promenljivi momenat ali dodatno povećavamo kompleksnost sistema (2 ulaza i 1 izlaz), merenje momenta može biti neizvodljivo.
- Brzinski servo mehanizam omogućava da se motor okreće željenom brzinom pri promeni opterećenja. Informaciju o brzini motora dobijamo pomoću enkodera ili tahogeneratora, i proprocionalna je naponu na rotoru.

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Pogonski mehanizam

- Problem DC motora: velika brzina, mali momenat.
- Mehaničkim reduktorom rešavamo problem. Cilj ovog uređaja je da izvrši prilagođenje opterećenja motora kao i prilagođenje brzine konkretnoj aplikaciji.
- Moment se povećava N puta, a brzina smanjuje N puta.

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Hvala!

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