Innovative Teaching Approaches in development of Software Designed Instrumentation and its application in real-time systems

# Theory of Robotics Systems

## Localization – Kalman Filtar

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Itasdi

# Innovative Teaching Approaches in development of Software Designed Instrumentation and its application in real-time systems

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Innovative Teaching Approaches in development of Software Designed Instrumentation and its application in real-time systems

# Teorija Robotskih Sistema

## Localizacija

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# Loklazacija Kalmanovim filtrom (ukratko)

1. Predikcija (ACT) zasniva se na prethodnoj proceni i odometriji.
2. Opservacija (SEE) koristeći postojeće senzore na robotu.
3. Predikcija merenja zasnovana na predikciji i mapi.
4. Uparivanje opserviranih obeležja i mape.
5. Estimacija → Popravka pozicije ( posteriorna pozicija)

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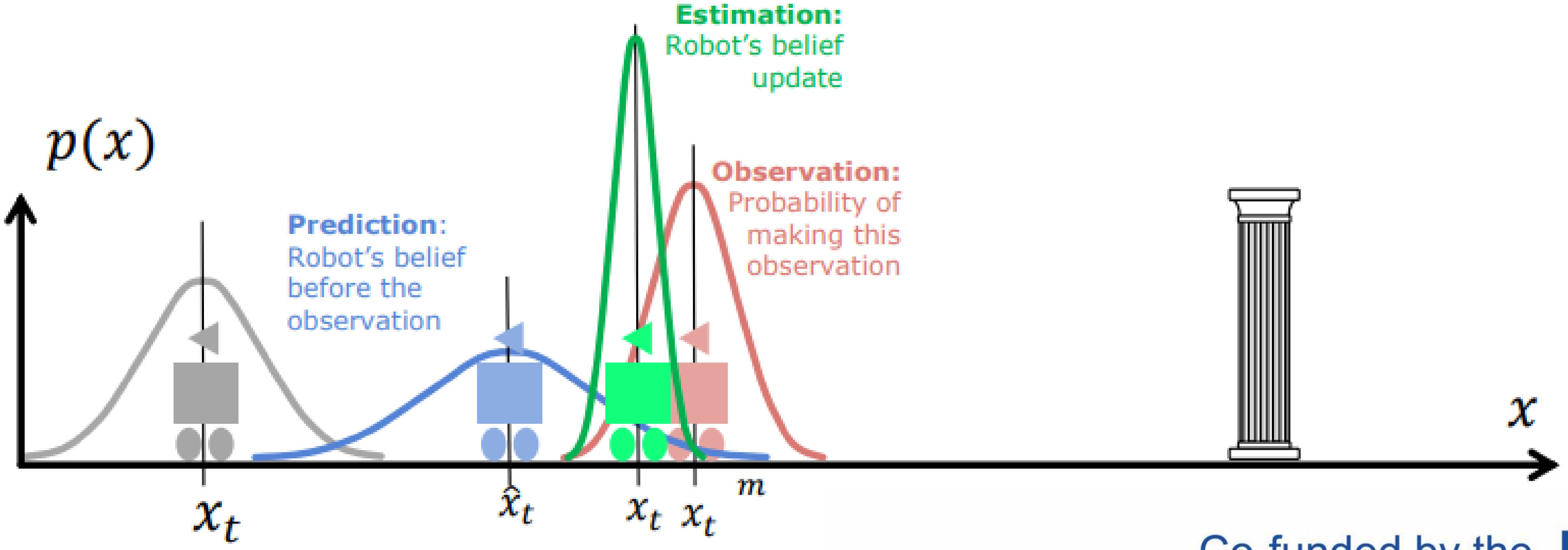
# Loklazacija Kalmanovim filtrom (ukratko)



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# Kalmanov filtar – teorija

- Sistem je linearan i model šuma može biti predstavljen kao Gausovski.
- Tokom faze predikcije i opservacije u celom sistemu se menjaju samo parametri Gausove raspodele $(\mu_t, \Sigma_t)$.
- Zasniva se na 4 jednačine:
    - 2 za estimaciju $(\mu_t, \Sigma_t)$ u fazi predikcije.
    - 2 za popravku $(\mu_t, \Sigma_t)$ u fazi opservacije.
- Koristi teoremu o totalnoj verovatnoći i Bajesovo pravilo.

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# Kalmanov filtar – Teorema o totalnoj verovatnoći

- Posmatramo dve nezavisne slučajne promenljive $x_1$ i $x_2$ koje normalno raspoređene.
- $x_1 = N(\mu_1, \sigma_1^2), x_2 = N(\mu_2, \sigma_2^2)$
- Postoji funkcija $y = f(x_1, x_2)$ - $f$ linearna funkcija $y = Ax_1 + Bx_2$
- Kako će biti raspodeljeno $y$? $< y > = A\mu_1 + B\mu_1; \sigma_y^2 = A^2\sigma_1^2 + B^2\sigma_2^2$
- U slušaju vektora: $< y > = A\mu_1 + B\mu_1; \Sigma_y = A\Sigma_1^2 A^T + B\Sigma_2^2 B^T$

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# Kalmanov filtar – Teorema o totalnoj verovatnoći

- Šta ako $f$ nije linearna funkcija? $y$ nije normalno raspoređeno
- Linearizacije prvog reda: $y \cong f(\mu_1, \mu_2) + F_{x_1}(x_1 - \mu_1) + F_{x_2}(x_2 - \mu_2)$
- $F_{x_1}$ i $F_{x_2}$ predstavljaju Jakobijane od $f$
- $< y > = y \cong f(\mu_1, \mu_2);$
- $\Sigma_y = F_{x_1}\Sigma_1^2 F_{x_1}^T + F_{x_2}\Sigma_2^2 F_{x_2}^T$

Predikcija za EKF

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# Kalmanov filtar – Bajesovo pravilo

- Pozicija robota $q$, verovatnoća da se robot nalazi na toj poziciji $p_1(q)$ i verovatnoća da se robot nalazi na toj poziciji prema eksteroceptivnim senzorima $p_2(q)$.
- $p_1(q) = N(\hat{q}_1, \sigma_1^2); p_2(q) = N(\hat{q}_2, \sigma_2^2)$
- Finalna raspodela $p(q)$ će biti proporcionalna proizvodu $p_1(q) \cdot p_2(q)$

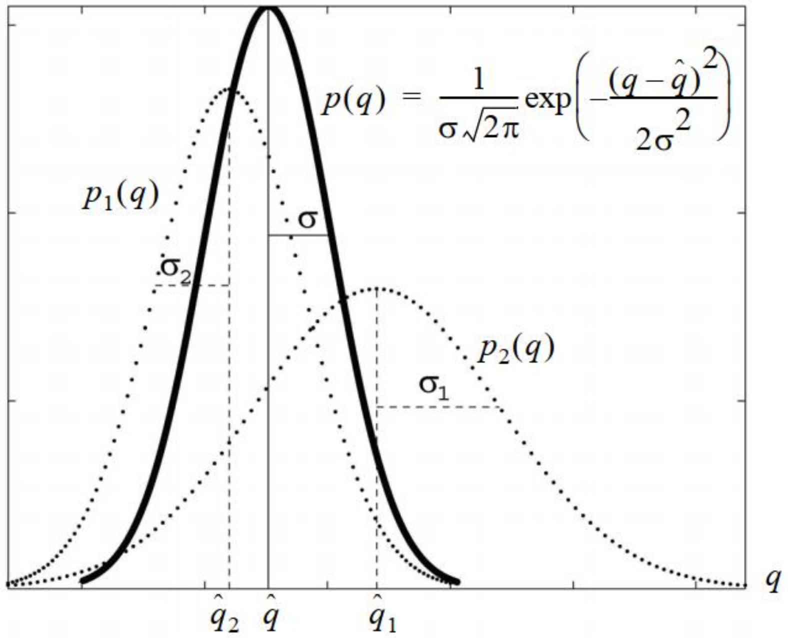
$$\frac{1}{\sigma_1\sqrt{2\pi}}\exp\left(-\frac{(q-\hat{q}_1)^2}{2\sigma_1{}^2}\right)\cdot\frac{1}{\sigma_2\sqrt{2\pi}}\exp\left(-\frac{(q-\hat{q}_2)^2}{2\sigma_2{}^2}\right)=\frac{1}{\sigma_1\sigma_2 2\pi}\exp\left(-\frac{(q-\hat{q}_1)^2}{2\sigma_1{}^2}-\frac{(q-\hat{q}_2)^2}{2\sigma_2{}^2}\right)$$

# Kalmanov filtar – Bajesovo pravilo

$$\frac{1}{\sigma_1\sqrt{2\pi}}\exp\left(-\frac{(q-\hat{q}_1)^2}{2\sigma_1^2}\right)\cdot\frac{1}{\sigma_2\sqrt{2\pi}}\exp\left(-\frac{(q-\hat{q}_2)^2}{2\sigma_2^2}\right) =$$

$$\frac{1}{\sigma_1\sigma_2 2\pi}\exp\left(-\frac{(q-\hat{q}_1)^2}{2\sigma_1^2}-\frac{(q-\hat{q}_2)^2}{2\sigma_2^2}\right) = \Omega\exp\left(-\frac{(q-\hat{q})^2}{2\sigma^2}\right)$$

$$\hat{q} = \hat{q}_1 + \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}(\hat{q}_2-\hat{q}_1) \qquad \sigma^2 = \sigma_1^2 - \frac{\sigma_1^4}{\sigma_1^2+\sigma_2^2}$$



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# Kalmanov filtar

- Ukoliko je u pitanju više-dimenziona slučajna promenljiva
- $\hat{q} = q_1 + P(P + R)^{-1}(q_2 - q_1)$
- $\hat{P} = P - P(P + R)^{-1}P$
- Gde su $P$ i $R$ kovarijacione matrice slučajnih promenljivih $q_1$ i $q_2$
- $K = P(P + R)^{-1}$ Kalmanovo pojačanje
- Inovacija - $q_2 - q_1$; Kovarijacona matrica inovacije - $\sum_{IN} = P + R$

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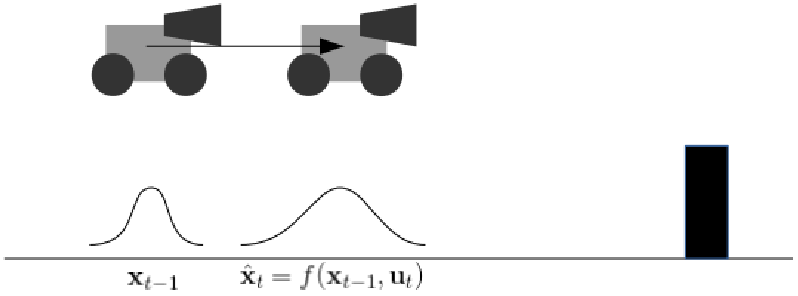
# Primena Kalmanovog filtar na AMR

- Predikcija:
    - Pozicija $x_t$ u trenutku $t$ zavisi od pozicije u prethodnom trenutku $x_{t-1}$ i kretanja robota koje se desi nakon upravljačkog signala $\boldsymbol{u}_t$.
    - $\hat{x}_t = f(x_{t-1}, \boldsymbol{u}_t); \boldsymbol{P}_t = \boldsymbol{F}_x \boldsymbol{P}_{t-1} \boldsymbol{F}_x^T + \boldsymbol{F}_u \boldsymbol{Q}_t \boldsymbol{F}_u^T$
    - $Q_t = \begin{bmatrix} k|\Delta s_l| & 0 \\ 0 & k|\Delta s_r| \end{bmatrix}$



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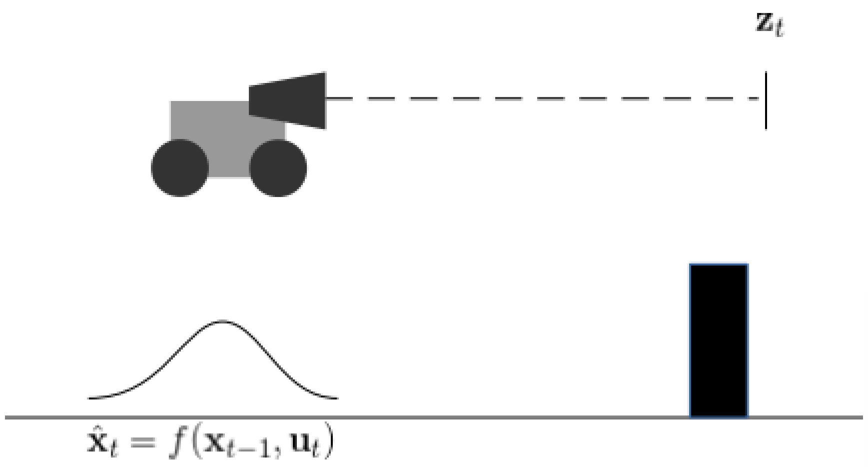
# Primena Kalmanovog filtar na AMR

- Opservacija:
  - Merenja senzora $z_t$ u trenutku $t$. U opštem slučaju postoji $n$ merenja $z_n^i$ $(i = 1 \dots n)$
  - Transformacija obeležja mape u lokalni koordinatni sistem se vrši funkcijom $h$.



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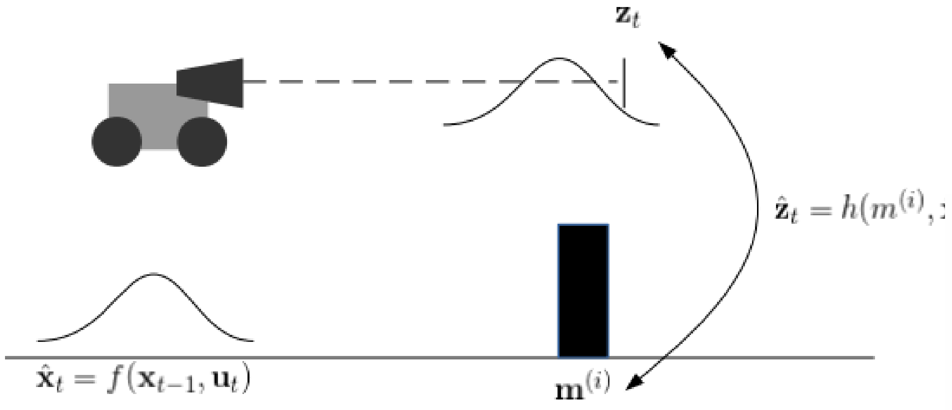
# Primena Kalmanovog filtar na AMR

- Predikcija merenja:
  - Predviđena pozicija robota $\hat{x}_t$ i obeležja mape $M$ koristimo za predviđanje merenja $z_t^j$
  - $\hat{z}_t^j = h^j(\hat{x}_t, m^j)$
  - $m^j = [\alpha^i \quad r^i]^T$ - jedno obeležje mape

$$\mathbf{H} = \begin{bmatrix} \frac{\partial h_1}{\partial x} & \frac{\partial h_1}{\partial y} & \frac{\partial h_1}{\partial \theta} \\ \frac{\partial h_2}{\partial x} & \frac{\partial h_2}{\partial y} & \frac{\partial h_2}{\partial \theta} \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & -1 \\ -\cos({}^W\alpha^j) & -\sin({}^W\alpha^j) & 0 \end{bmatrix}$$



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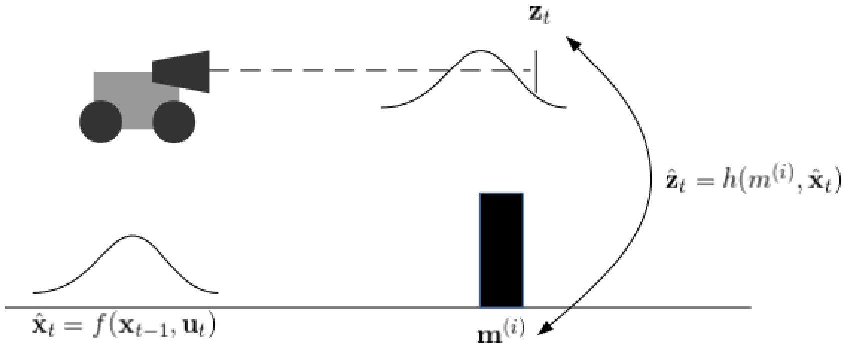
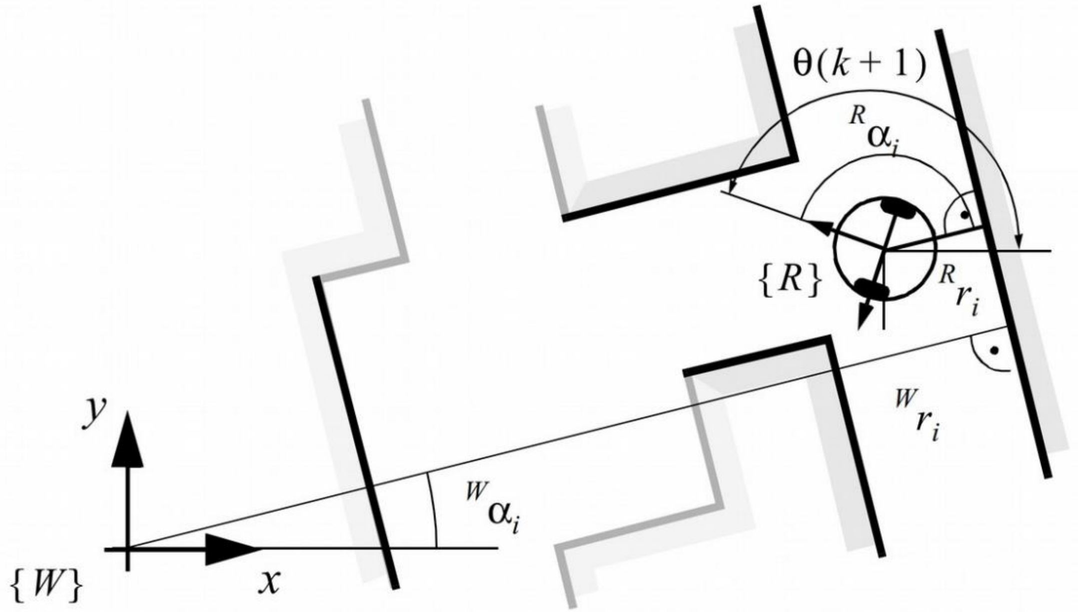
# Primena Kalmanovog filtar na AMR

- Predikcija merenja:
  - $m^j = [\alpha^i \quad r^i]^T$ - jedno obeležje mape
  - $\hat{z}_t^j = \begin{bmatrix} {}^R\hat{\alpha}^j \\ {}^R\hat{r}^j \end{bmatrix} = h^j(\hat{x}_t, m^j) =$
  - $\begin{bmatrix} {}^W\alpha^j - \hat{\theta}_t \\ {}^W r^j - (\hat{x}_t \cos({}^W\alpha^j) + \hat{y}_t \sin({}^W\alpha^j)) \end{bmatrix}$





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# Primena Kalmanovog filtar na AMR

- Uparivanje:
  - Za uparivanje koristimo razliku između svih merenja i mape.
  - Ovakve razlike nazivamo *inovacijama*
  - $v_t^{ij} = z_t^j - \hat{z}_t^i$ ← Predikcija merenja

Inovacija Opservacija

  - $\Sigma_{IN_t}^{ij} = \widehat{H}_t^i \hat{P}_t (\widehat{H}_t^i)^T + R_t^j$ ← Kovarijansa merenja

Kovarijansa inovacije

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# Primena Kalmanovog filtar na AMR

- Uparivanje:
  - Da bi za obeležje sa mape mogli da uparimo sa opserviranim obeležjem mora biti zadovoljen određeni uslov.
  - Najčešće se koristi Mahalonobisova distanca
  - $d_t^{ij} = \left(v_t^{ij}\right)^T \left(\Sigma_{IN_t}^{ij}\right)^{-1} v_t^{ij}$
  - $d_t^{ij} < g^2$ - validaciona granica

# Primena Kalmanovog filtar na AMR

- Estimacija:
  - Kada imamo predikciju i kada smo izvršili uparivanje obeležja primenjujemo Bajesovo pravilo.
  - $x_t = \hat{x}_t + K_t v_t$
  - $K_t = \hat{P}_t \hat{H}_t^T \left(\sum_{IN_t}\right)^{-1}$
  - $P_t = \hat{P}_t - K_t \sum_{IN_t} K_t^T$

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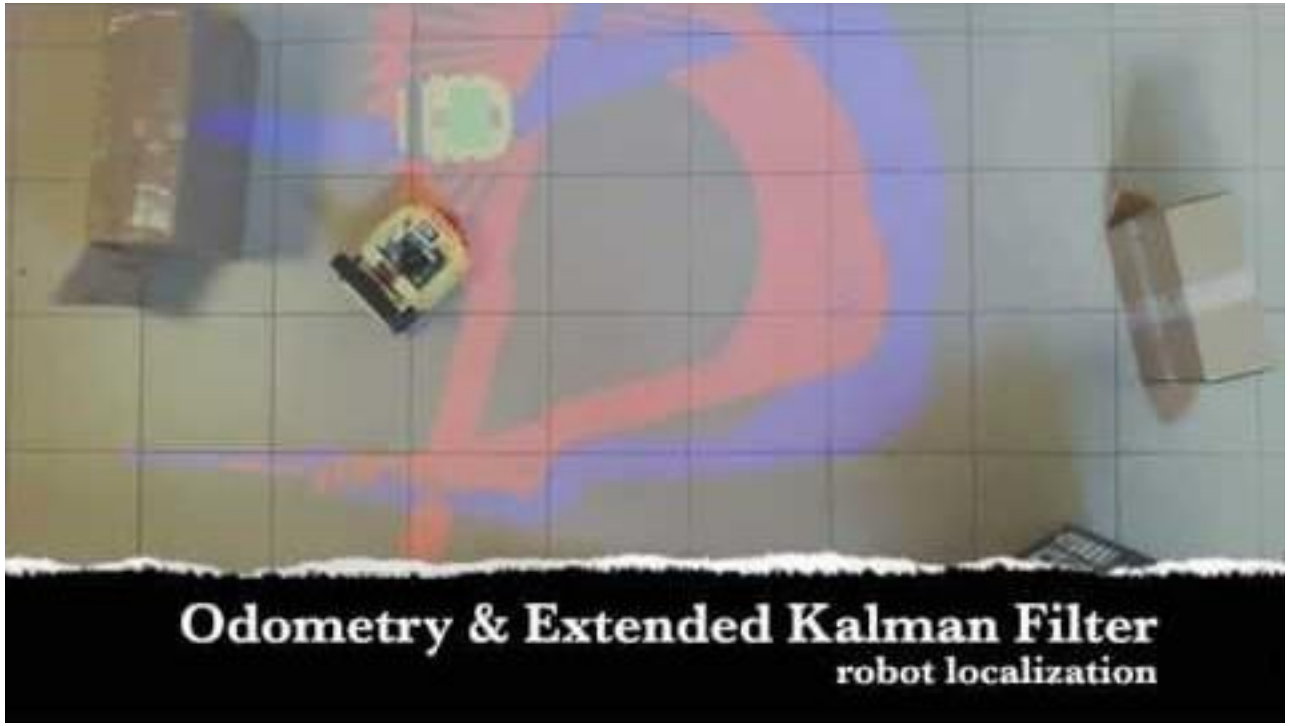
# Primena Kalmanovog filtar na AMR



https://www.youtube.com/watch?v=TnCouG221uo

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# Thanks!

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