Itasdi

Innovative Teaching Approaches in development of Software Designed Instrumentation and its application in real-time systems

Theory of Robotics Systems

Line Fitting – Uncertainty

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Innovative Teaching Approaches in development of Software Designed Instrumentation and its application in real-time systems

Faculty of Technical Sciences

Ss. Cyril and Methodius University
Faculty of Electrical Engineering and Information Technologies

Zagreb University of Applied Sciences

School of Electrical Engineering
University of Belgrade

Faculty of Physics
Warsaw University of Technology

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Reprezentacija nesigurnosti merenja

- Opažanje u realnom okruženju je uvek nesigurno.
  - Kako možemo predstaviti i kvantifikovati nesigurnost?
  - Kako se propagira nesigurnost?
  - Kako se ovo sve odnosi na mobilnu robotiku?

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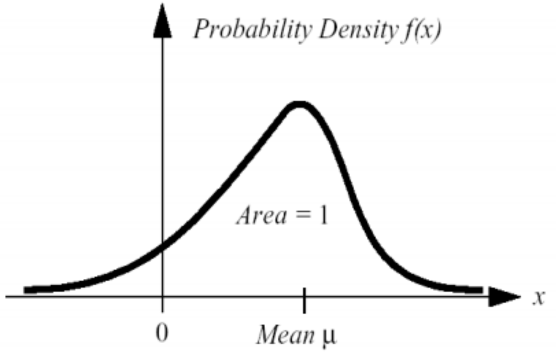
Reprezentacija nesigurnosti merenja

- Koristi se Funkcionalna Gustina Verovatnoće (FGV) za opisivanje karakteristika promenljive x:



$$\int_{-\infty}^{\infty} f(x)dx = 1$$

- Očekivana vrednost promenljive: $\mu = E[X] = \int_{-\infty}^{\infty} xf(x)dx$
- Varijansa promenljive: $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

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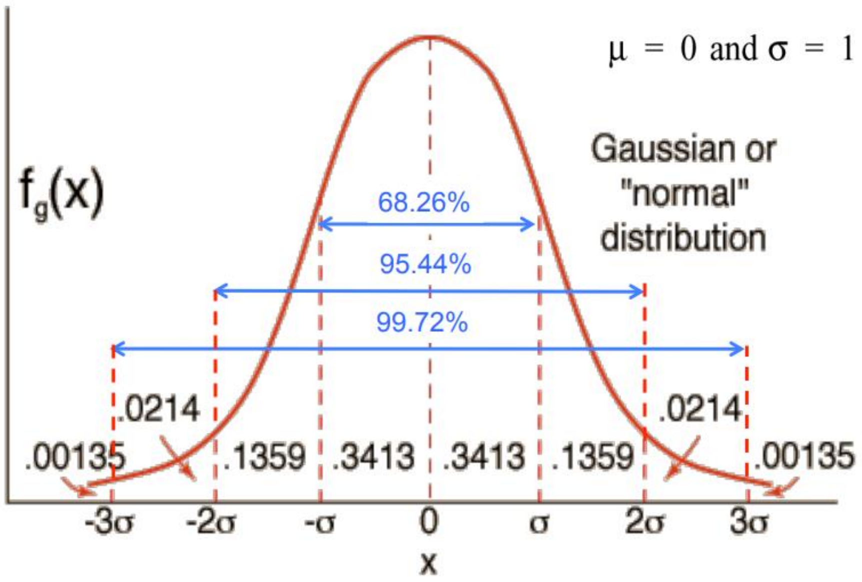
Gausovska FGV

- Najčešće korišćena FGV za krakterizaciju nesigurnosti je Gausovska.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



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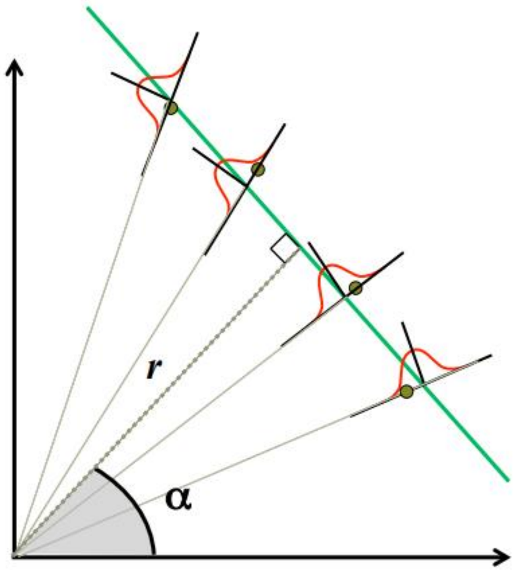
Zakon propagiranja greške

- Posmatramo problem izdvajanja linije baziranom na merenjima koja imaju određenu nesigurnost.
- Model parametara je predstavljen u polarnim koordinatama.
- Koja je nesigurnost za izdvojenu liniju ukoliko znamo nesigurnost tačaka koje čine tu liniju?



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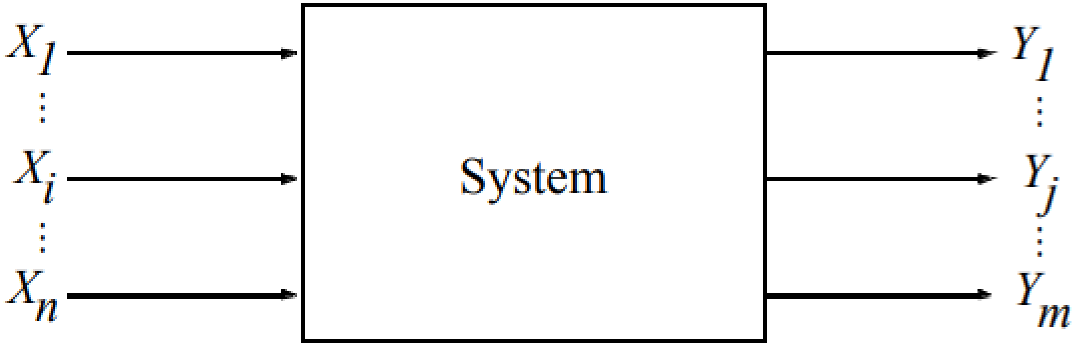
Zakon propagiranja greške

- Propagacija greške je sistem sa više ulaza i više izlaza.



$$Y_j = f_j(X_1 \ldots X_n)$$

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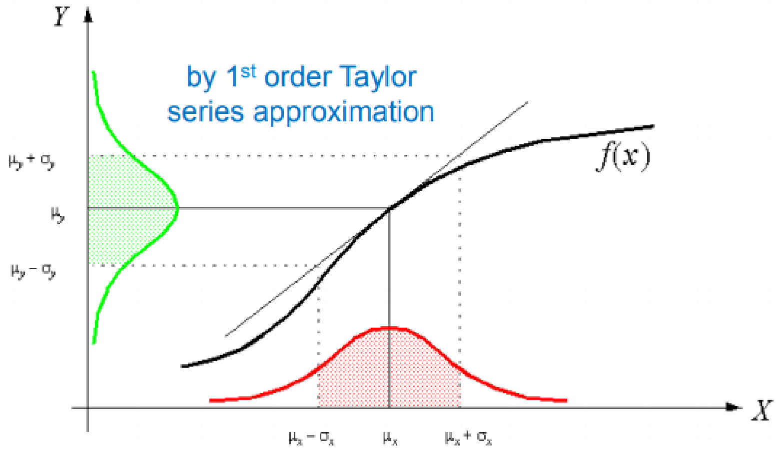
Zakon propagiranja greške

- 1D slučaj nelinearne propagacije greške.
- Može se pokazati da je izlazna kovarijaciona matrica: $C_{YY} = F_{YX} C_{XX} F_{XY}$ $, F_{XY} = F_{YX}^{T}$
- Gde je:
  - C_{XX} kovarijaciona matrica ulaznih nesigurnosti
  - C_{YY} kovarijaciona matrica propagiranih nesigurnosti
  - F_{YX} Jacobian matrica

$$F_{YX} = \begin{pmatrix} \frac{\partial f_1}{\partial X_1} & \cdots & \frac{\partial f_m}{\partial X_1} \\ \vdots & \cdots & \vdots \\ \frac{\partial f_m}{\partial X_1} & \cdots & \frac{\partial f_m}{\partial X_n} \end{pmatrix}$$



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Zakon propagiranja greške – izdvajanje linije

- Rastojanje tačke od linije

$$\rho_i \cos(\theta_i - \alpha) - r = d_i$$

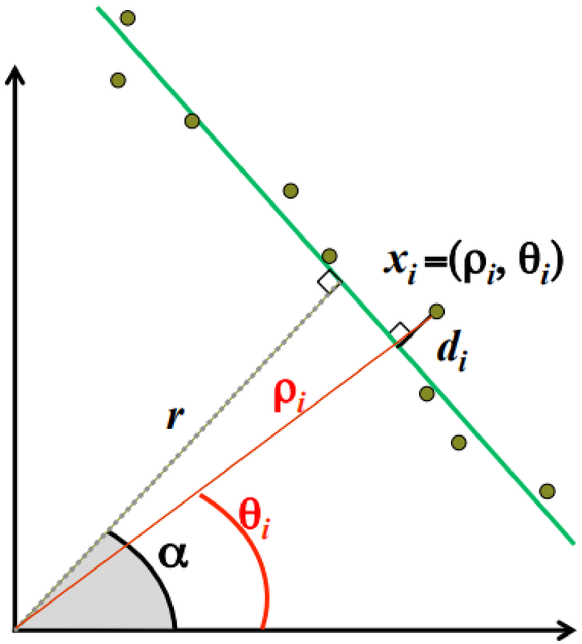
- Ukoliko pretpostavimo da su sva merenja podjednako nesigurna

$$S = \sum_i d_i^2 = \sum_i (\rho_i \cos(\theta_i - \alpha) - r)^2$$

- Cilj: minimizovati **S** da dobijemo što tačnije (*r*,*a*):

$$\frac{\partial S}{\partial \alpha} = 0 \qquad \frac{\partial S}{\partial r} = 0$$

- Unweighted Least Squares



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Zakon propagiranja greške – izdvajanje linije

- Rastojanje tačke od linije

$$\rho_i \cos(\theta_i - \alpha) - r = d_i$$

- Merenja mogu imati jedinstvene nesigurnosti

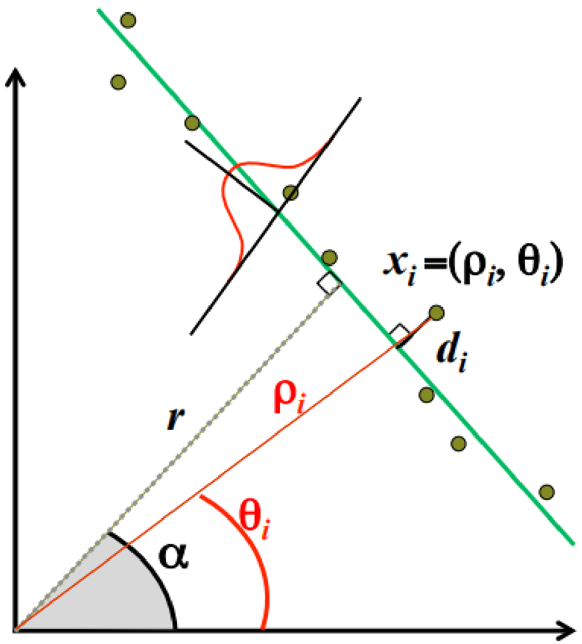
$$S = \sum w_i d_i^2 = \sum w_i (\rho_i \cos(\theta_i - \alpha) - r)^2$$

$$w_i = 1/\sigma_i^2$$

- Cilj: minimizovati **S** da dobijemo što tačnije (*r*,*a*):

$$\frac{\partial S}{\partial \alpha} = 0 \qquad \frac{\partial S}{\partial r} = 0$$

- Weighted Least Squares



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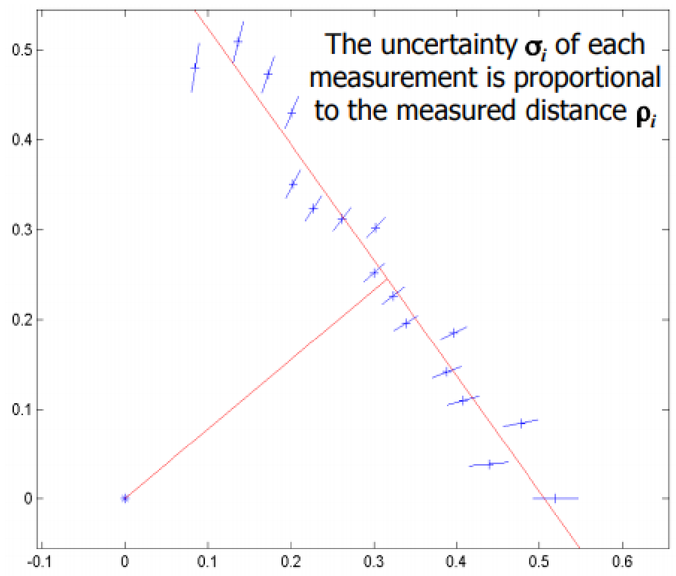
Zakon propagiranja greške – izdvajanje linije

- Rešavamo sistem: $\frac{\partial S}{\partial \alpha} = 0 \qquad \frac{\partial S}{\partial r} = 0$
- Dobijamo parametre linija:

$$\alpha = \frac{1}{2}\operatorname{atan}\left(\frac{\sum w_i \rho_i^2 \sin 2\theta_i - \frac{2}{\sum w_i}\sum\sum w_i w_j \rho_i \rho_j \cos\theta_i \sin\theta_j}{\sum w_i \rho_i^2 \cos 2\theta_i - \frac{1}{\sum w_i}\sum\sum w_i w_j \rho_i \rho_j \cos(\theta_i + \theta_j)}\right) \qquad r = \frac{\sum w_i \rho_i \cos(\theta_i - \alpha)}{\sum w_i}$$



- Ako svaki od ulaznih parametra ima svoju nesigurnost kako predstaviti nesigurnost linije?

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Zakon propagiranja greške – izdvajanje linije

- Nesigurnost svakog merenja je predstavljena kovarijacionom matricom
- Nesigurnost linije je predstavljena svojom kovarijacionom matricom

$$C_{xx_i} = \begin{bmatrix} \sigma_{\rho_i}^2 & 0 \\ 0 & \sigma_{\theta_i}^2 \end{bmatrix}$$

$$C_{ll} = \begin{bmatrix} \sigma_{\alpha}^2 & \sigma_{\alpha r} \\ \sigma_{r\alpha} & \sigma_{r}^2 \end{bmatrix} = ?$$

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Zakon propagiranja greške – izdvajanje linije

- Kovarijacionu matricu svih merenja definišemo kao:

$$C_{xx} = \begin{bmatrix} diag(\sigma_\rho^2) & 0 \\ 0 & diag(\sigma_\theta^2) \end{bmatrix} = \begin{bmatrix} \dots & 0 & 0 & . & 0 & 0 & . \\ . & \sigma_{\rho_i}^2 & 0 & . & 0 & 0 & . \\ . & 0 & \sigma_{\rho_{i+1}}^2 & . & 0 & 0 & . \\ . & . & . & \dots & . & . & . \\ . & 0 & 0 & . & \sigma_{\theta_i}^2 & 0 & . \\ . & 0 & 0 & . & 0 & \sigma_{\theta_i+1}^2 & . \\ . & . & . & . & . & . & \dots \end{bmatrix}_{2n \times 2n}$$

- Zakon propagacije greške:

$$C_{ll} = F_{lx} C_{xx} F_{xl}$$

$$F_{lx} = \begin{bmatrix} \dots & \frac{\partial \alpha}{\partial \rho_i} & \frac{\partial \alpha}{\partial \rho_{i+1}} & \dots & \frac{\partial \alpha}{\partial \theta_i} & \frac{\partial \alpha}{\partial \theta_{i+1}} & \dots \\ \dots & \frac{\partial r}{\partial \rho_i} & \frac{\partial r}{\partial \rho_{i+1}} & \dots & \frac{\partial r}{\partial \theta_i} & \frac{\partial r}{\partial \theta_{i+1}} & \dots \end{bmatrix}$$

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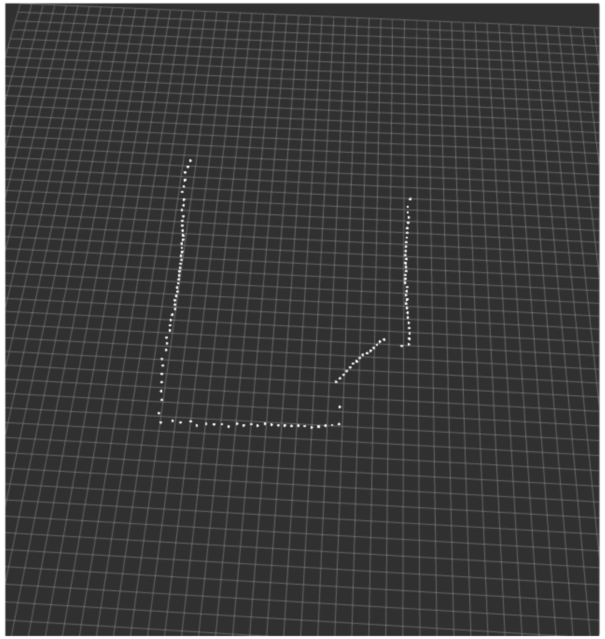
Izdvajanje linije iz oblaka tačaka

- Tri glavna problema:
  - Koliko linija se postoji?
  - Segmentacija: Koje tačke pripadaju kojoj liniji?
  - Fitovaje linije: Kako estimirati parametre linije?
- Najpoznatiji algoritmi:
  - Split-and-merge
  - Linearna regresija
  - RANSAC



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Split-and-Merge

- Popularan algoritam, originalno korišćen u kompjuterskoj viziji.
- Predstavlja rekurzivnu proceduru za izdvajanje i spajanje linja.

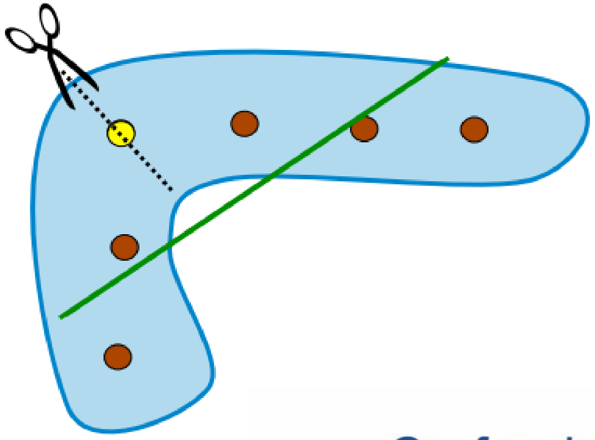
Let **S** be the set of all data points

Split

- Fit a line to points in current set **S**
- Find the most distant point to the line
- If distance > threshold ⇨ split set & repeat with left and right point sets

Merge

- If two consecutive segments are collinear enough, obtain the common line and find the most distant point
- If distance <= threshold, merge both segments



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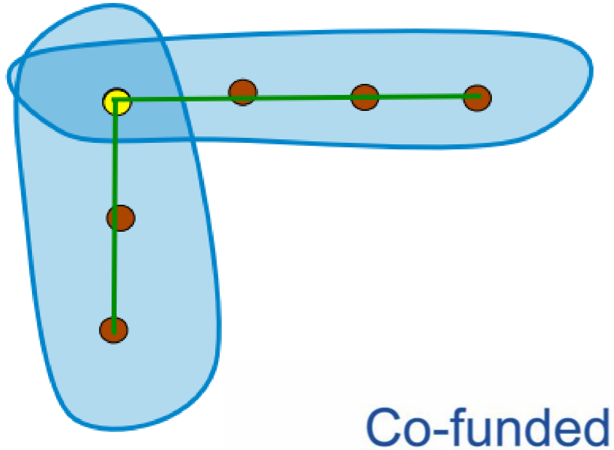

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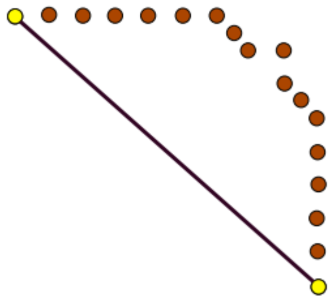
Split-and-Merge (iterativni)

- Povežu se prva i poslednja tačka



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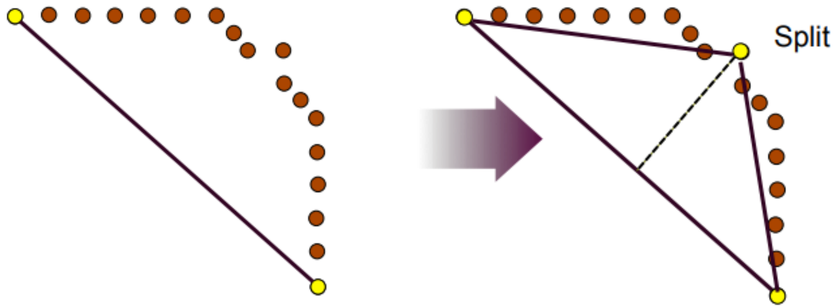

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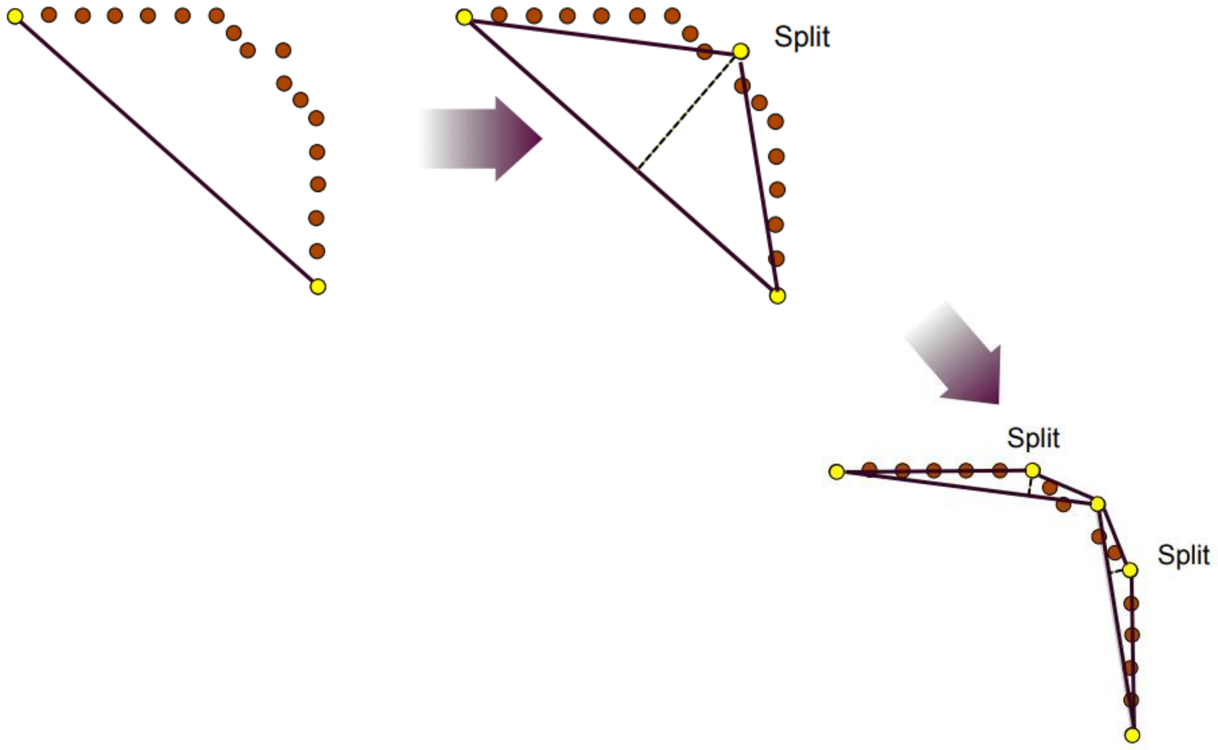

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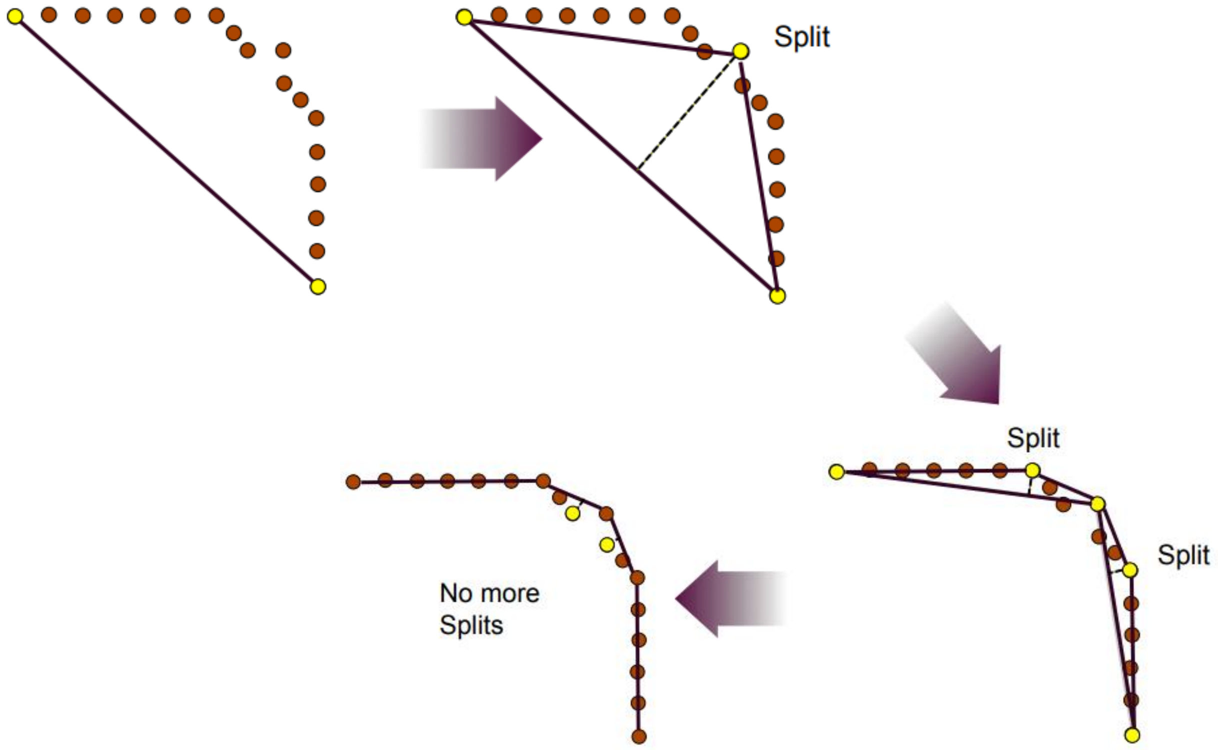

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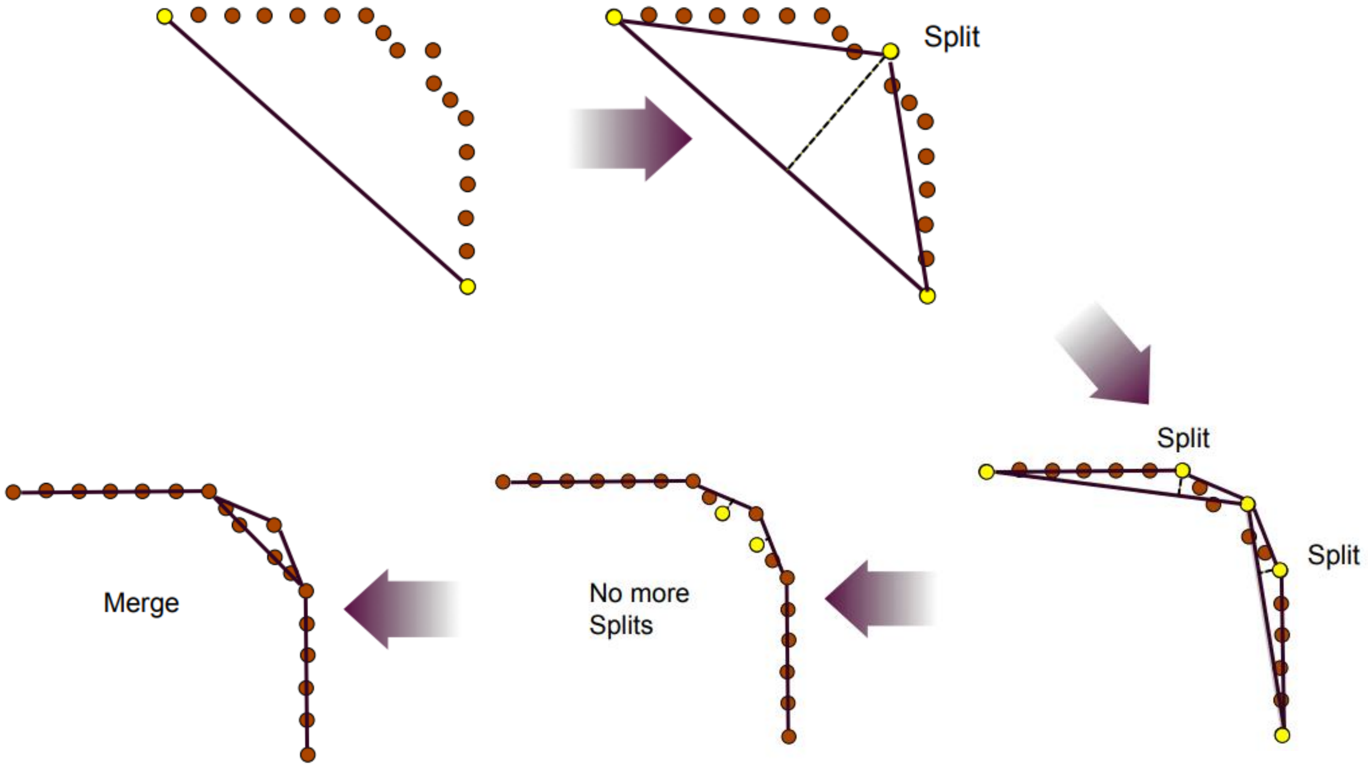

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Izdvajanje linija - poređenje

	Complexity	Speed (Hz)	False positives	Precision
Split-and-Merge	$N \log N$	1500	10%	+++
Incremental	$S N$	600	6%	+++
Line-Regression	$N N_f$	400	10%	+++
RANSAC	$S N k$	30	30%	++++
Hough-Transform	$S N N_C + S N_R N_C$	10	30%	++++
Expectation Maximization	$S N_1 N_2 N$	1	50%	++++

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Thanks!

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