Innovative Teaching Approaches in development of Software Designed Instrumentation and its application in real-time systems

Theory of Robotics Systems

Graph Search

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Innovative Teaching Approaches in development of Software Designed Instrumentation and its application in real-time systems

Faculty of Technical Sciences

Ss. Cyril and Methodius University
Faculty of Electrical Engineering and Information Technologies

Zagreb University of Applied Sciences

School of Electrical Engineering
University of Belgrade

Faculty of Physics
Warsaw University of Technology

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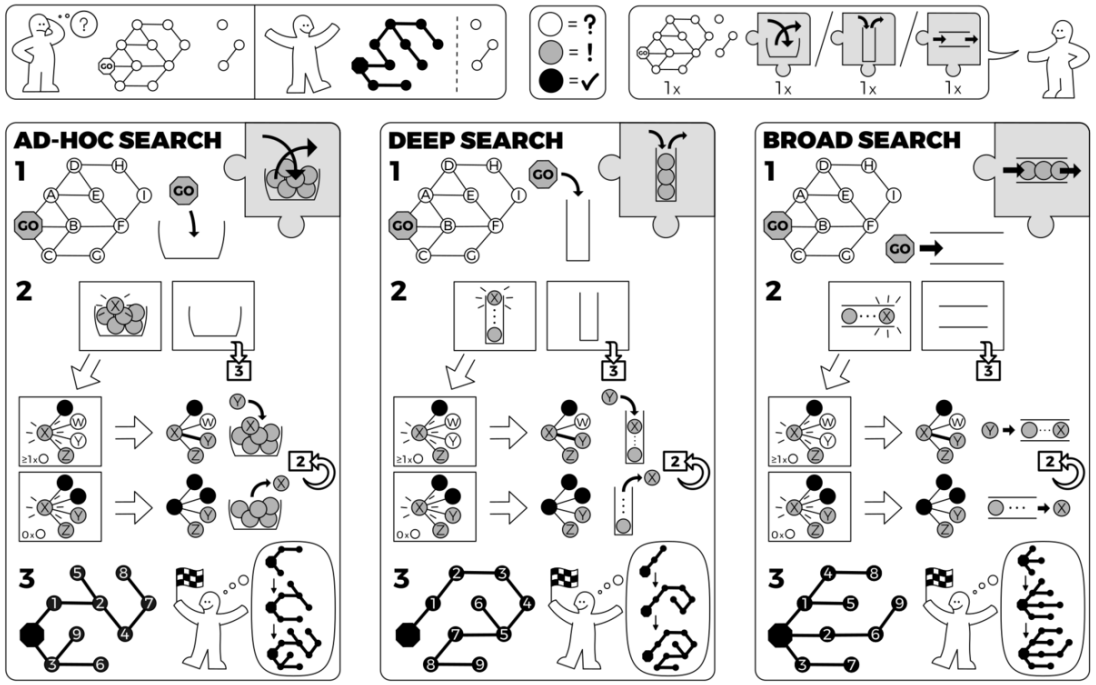
Metode pretrage grafova



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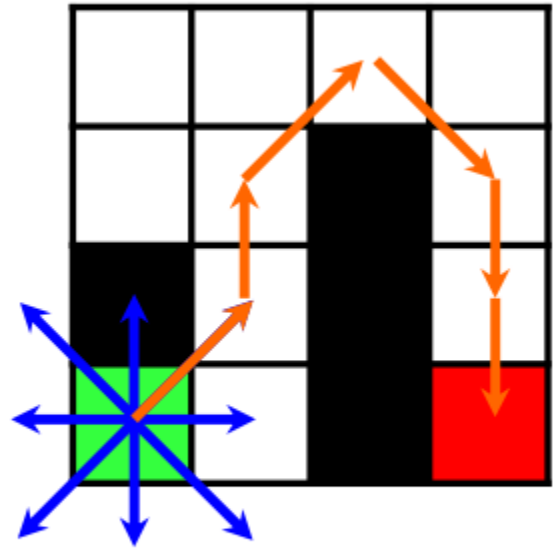
Primer 1

- Putanja od zelene ćelije do crvene
- 8 koraka iz jedne ćelije
- Koja je najkraća putanja?
- Šta ako imamo 4 moguća koraka iz ćelije.



Assumptions about robot motion

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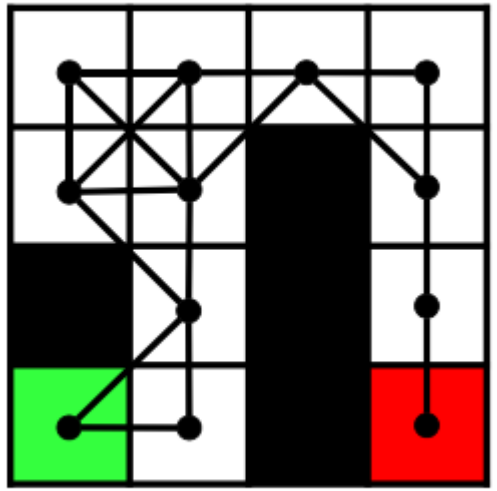
Primer 1

- Pretražimo dobijeni graf.
- Pretpostavljamo da sve ivice grafa mogu kreirati putanju.
- Rešimo problem tako što nađemo putanju od A do B.



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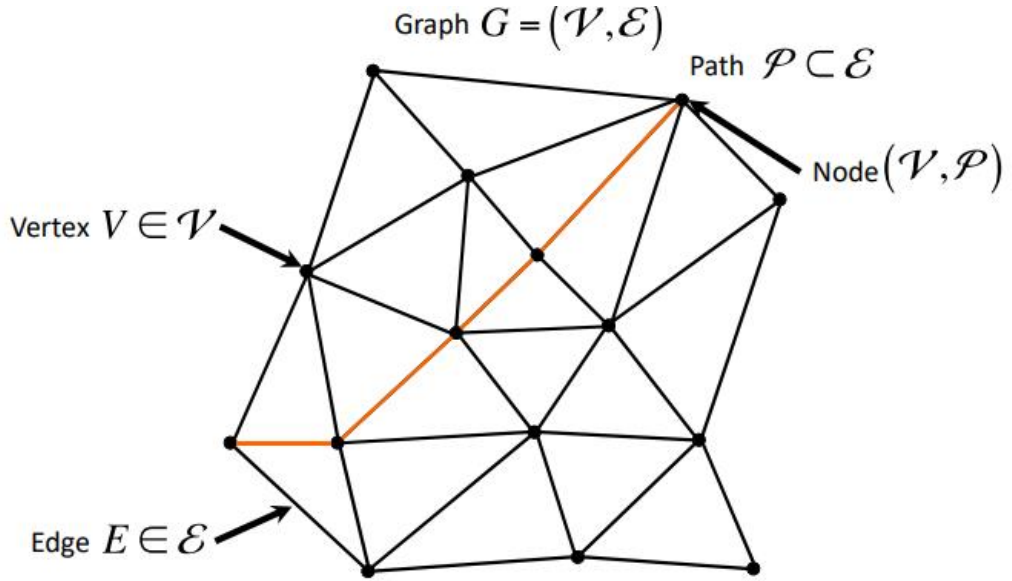
Pretraga grafova

- Ivice grafa mogu imati pridružene **težine**
  - Cena putanje
- Ima smisla da se razmatraju samo pozitivne težine.
- Algoritmi sa minimalnom cenom nikad ne posećuju isti čvor dva puta.
- Najpoznatije pretrage:
  - Pretraga u širinu (BFS)
  - Pretraga u dubinu (DFS)
  - Dijkstra's Algoritam
  - A*



Directed graph: edges have direction
Weighted graph: edges have costs

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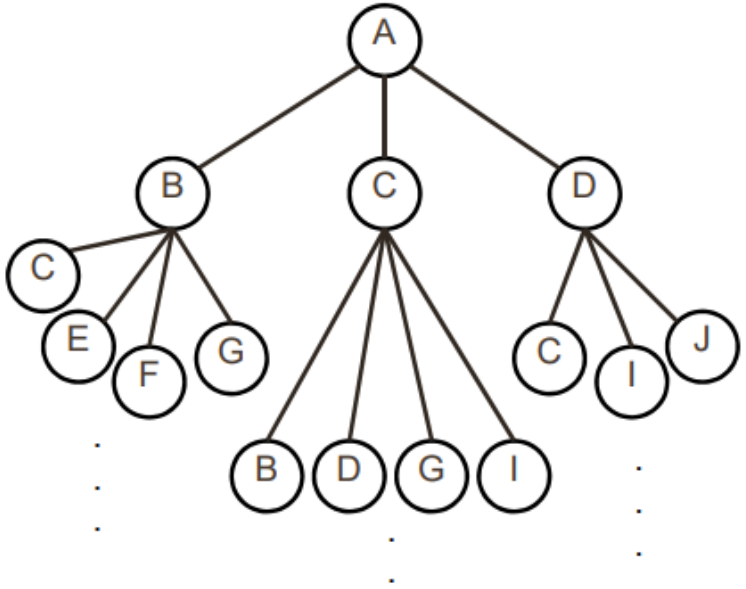
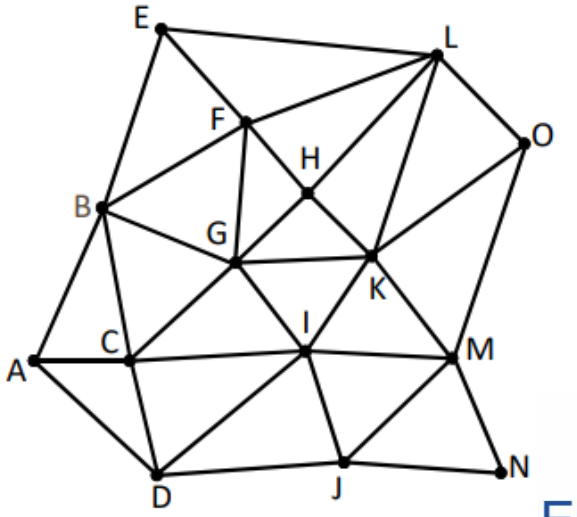
Drvo pretrage

- Konstruišemo „drvo“ pomoću kojeg vršimo pretragu.





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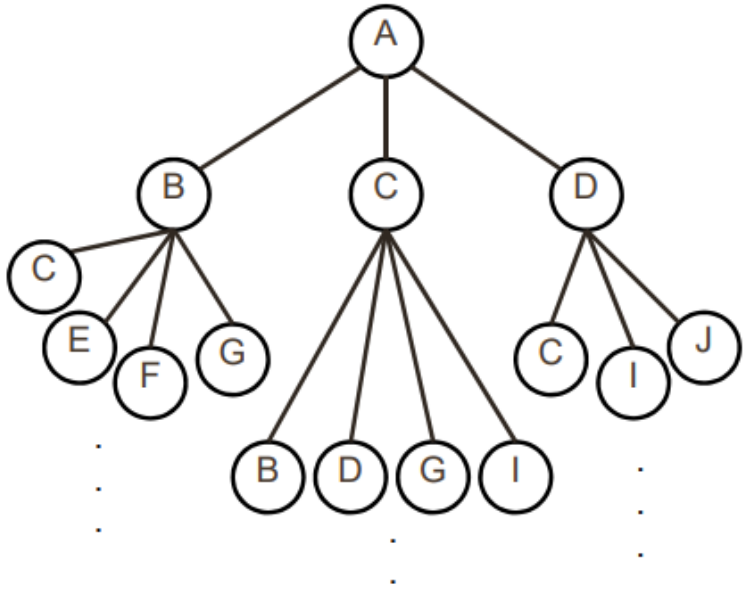
Drvo pretrage

- Drvo pretrage:
  - Početno stanje se uzima kao koren
  - Naslednici – skup svih mogućih akcija
  - Uglavnom želimo da izbegnemo konstrukciju kompletnog stabla



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Osnove pravolinijske pretrage

- Počnemo od korena (početno stanje), širimo drvo sve dok ne dođemo do cilja.
- Proširivanje nodova znači dodavanje njihovih naslednika.
- Treba probati da se nađe rešenje sa što manje proširivanja.
- Otvoren set – čvorovi iz kojih se mogu vršiti proširivanja
- Zatvoren set – svi oni čvorovi koje je algoritam posetio

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Osnove pravolinijske pretrage

```
FORWARD_SEARCH
1   Q.Insert(x_I) and mark x_I as visited
2   while Q not empty do
3       x ← Q.GetFirst()
4       if x ∈ X_G
5           return SUCCESS
6       forall u ∈ U(x)
7           x' ← f(x, u)
8           if x' not visited
9               Mark x' as visited
10              Q.Insert(x')
11          else
12              Resolve duplicate x'
13  return FAILURE
```

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Pretraga po širini

```
FORWARD SEARCH
1   Q.Insert(x_I) and mark x_I as visited
2   while Q not empty do
3       x ← Q.GetFirst()
4       if x ∈ X_G
5           return SUCCESS
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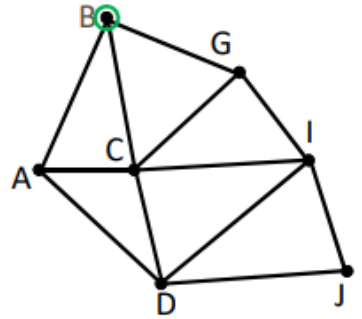

Open (Q): {B}

Closed: {}

Our (BFS) queue will be FIFO:
- push (*Q.Insert*) onto the end
- pop (*Q.GetFirst*) from the front

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Pretraga po širini

```
FORWARD_SEARCH
1   Q.Insert(x_I) and mark x_I as visited
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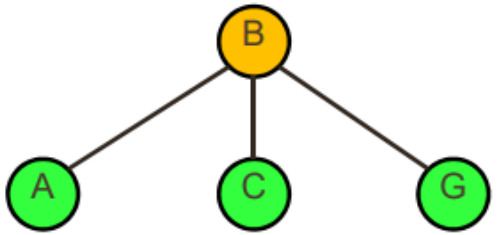

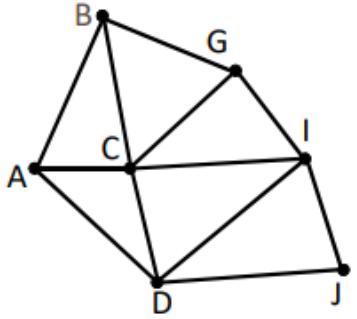

Open (Q): {A,C,G}

Closed: {B,A,C,G}

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Pretraga po širini

```
FORWARD_SEARCH
1   Q.Insert(x_I) and mark x_I as visited
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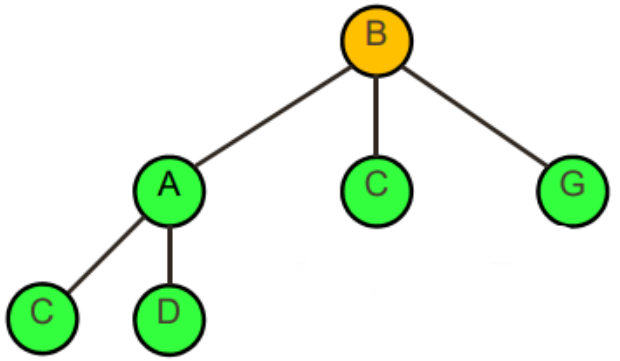

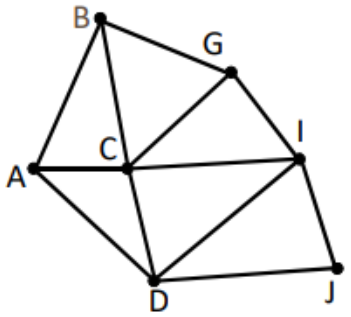

Open (Q): {C,G,D}

Closed: {B,A,C,G,D}

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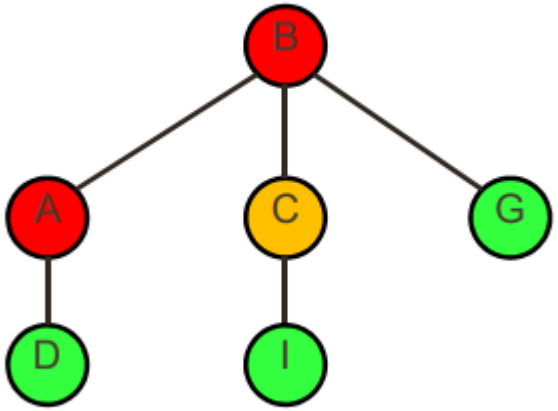
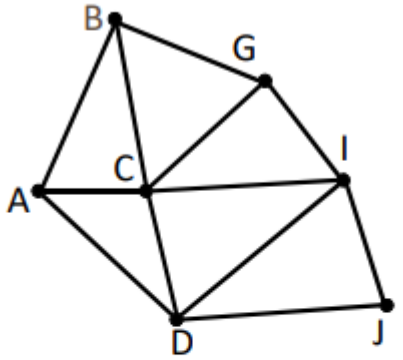
Pretraga po širini





Open (Q): {G,D,I}

Closed: {B,A,C,G,D,I}

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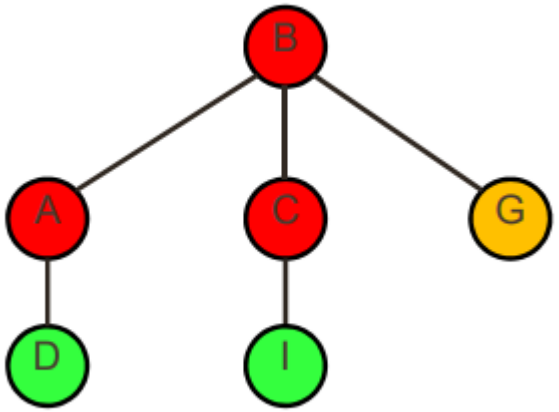
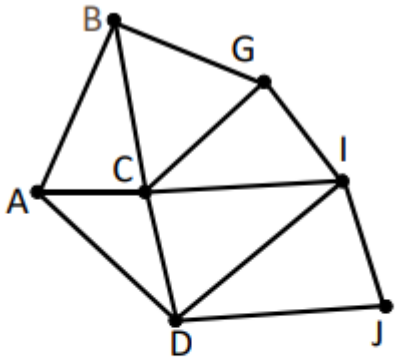
Pretraga po širini





Open (Q): {D,I}

Closed: {B,A,C,G,D,I}

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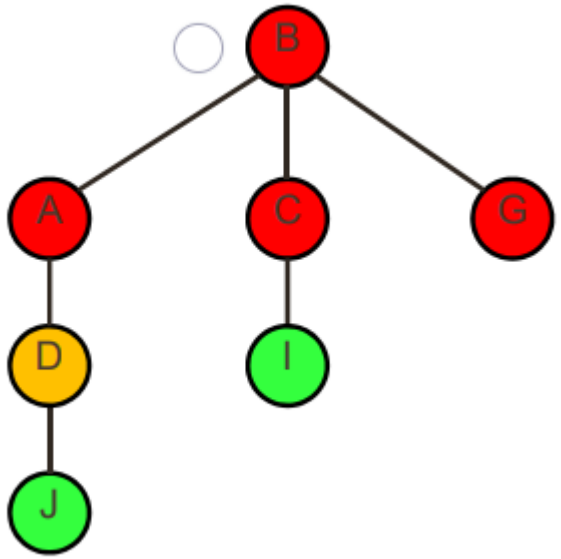
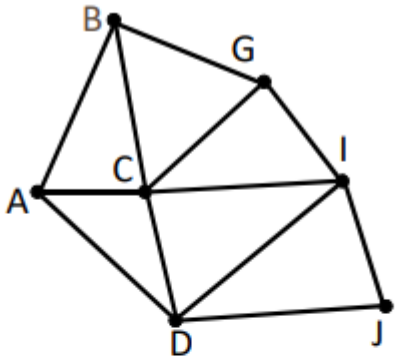
Itasdi

Pretraga po širini





Open (Q): {I,J}

Closed: {B,A,C,G,D,I,J}

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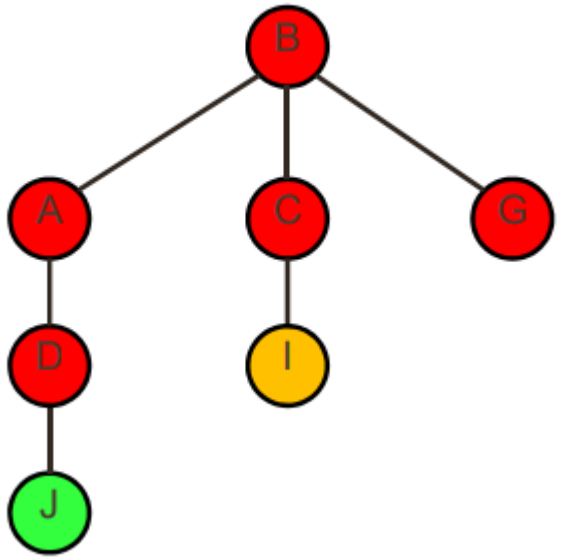
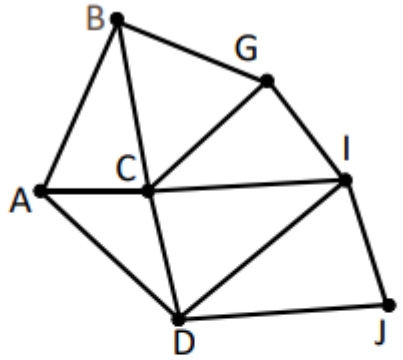
Pretraga po širini





Open (Q):	Closed:
{J}	{B,A,C,G,D,I,J}

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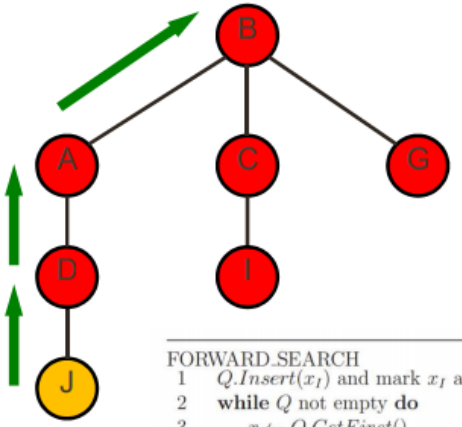
Itasdi

Pretraga po širini




```
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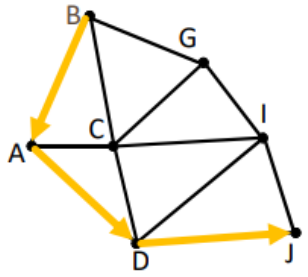

Open (Q): {}

Closed: {B,A,C,G,D,I,J}

Final path solution: B→A→D→J

Other solutions may exist but have the same number or more transitions

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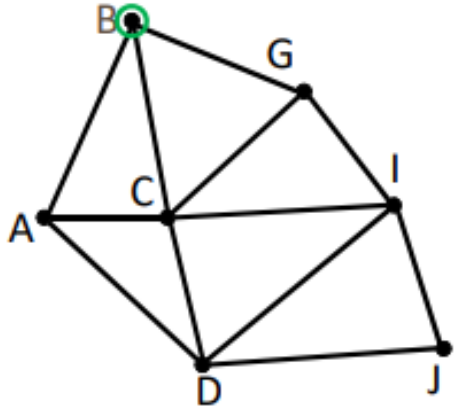
Itasdi

Pretraga po dubini

```
FORWARD SEARCH
1   Q.Insert(x_I) and mark x_I as visited
2   while Q not empty do
3       x ← Q.GetFirst()
4       if x ∈ X_G
5           return SUCCESS
6       forall u ∈ U(x)
7           x' ← f(x, u)
8           if x' not visited
9               Mark x' as visited
10              Q.Insert(x')
11          else
12              Resolve duplicate x'
13  return FAILURE
```



Open (Q): {B}

Closed: {}

Our (DFS) queue will be LIFO:
- push (*Q.Insert*) onto the front
- pop (*Q.GetFirst*) from the front

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Pretraga po dubini

```
FORWARD_SEARCH
1   Q.Insert(x_I) and mark x_I as visited
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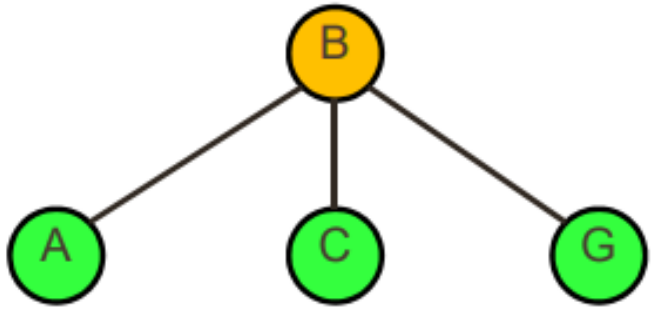
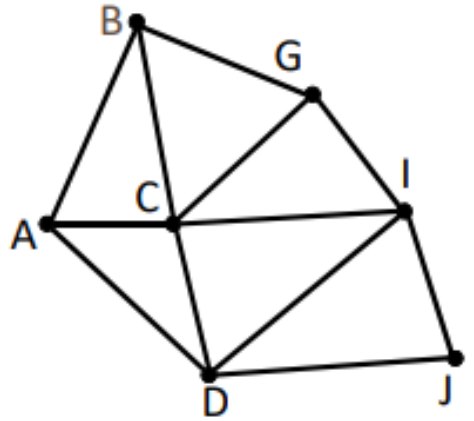
8           if x' not visited
9               Mark x' as visited
10              Q.Insert(x')
11          else
12              Resolve duplicate x'
13  return FAILURE
```





Open (Q): {A,C,G}

Closed: {B,A,C,G}

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Pretraga po dubini

```
FORWARD_SEARCH
1   Q.Insert(x_I) and mark x_I as visited
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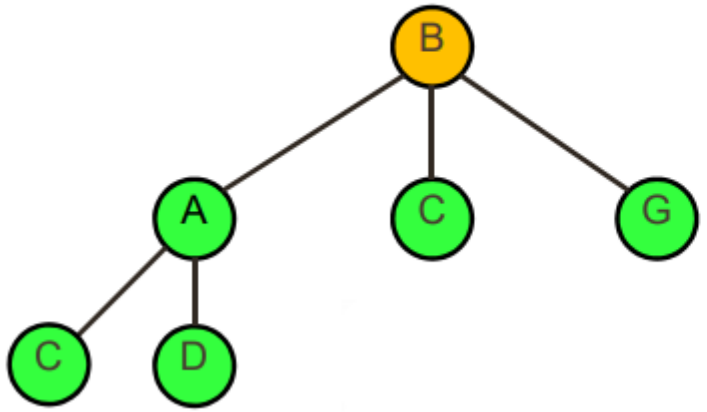

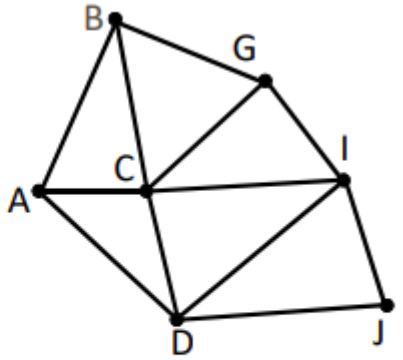

Open (Q): {D,C,G}

Closed: {B,A,C,G,D}

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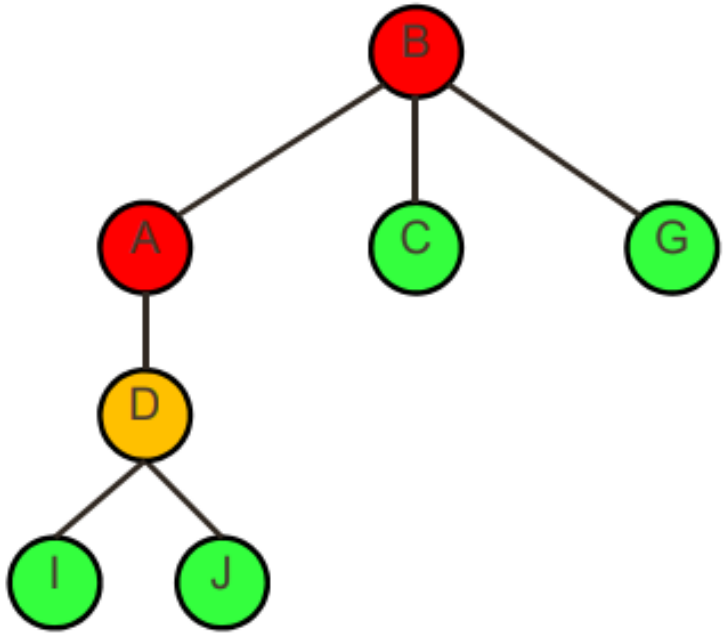
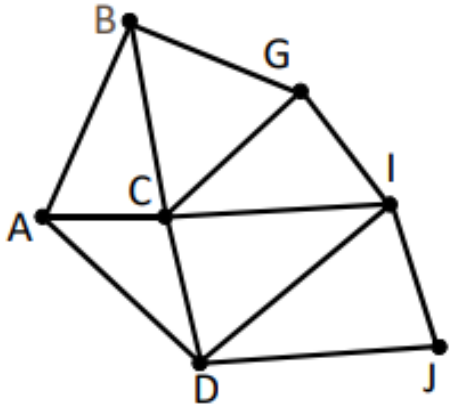
Pretraga po dubini





Open (Q): {I,J,C,G}

Closed: {B,A,C,G,D,I,J}

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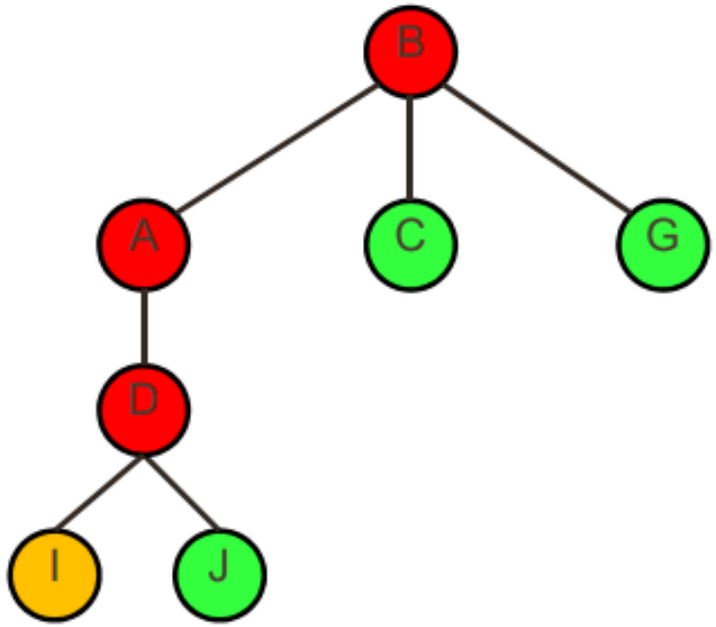
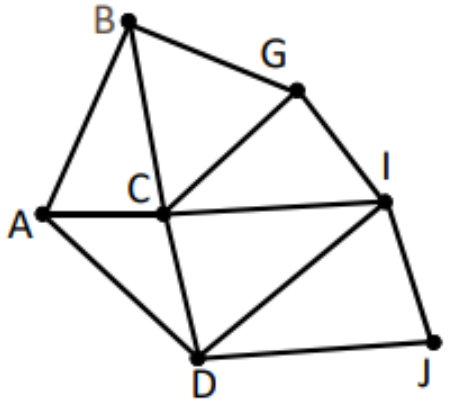
Pretraga po dubini





Open (Q): {J,C,G}

Closed: {B,A,C,G,D,I,J}

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Pretraga po dubini

```
FORWARD_SEARCH
1   Q.Insert(x_I) and mark x_I as visited
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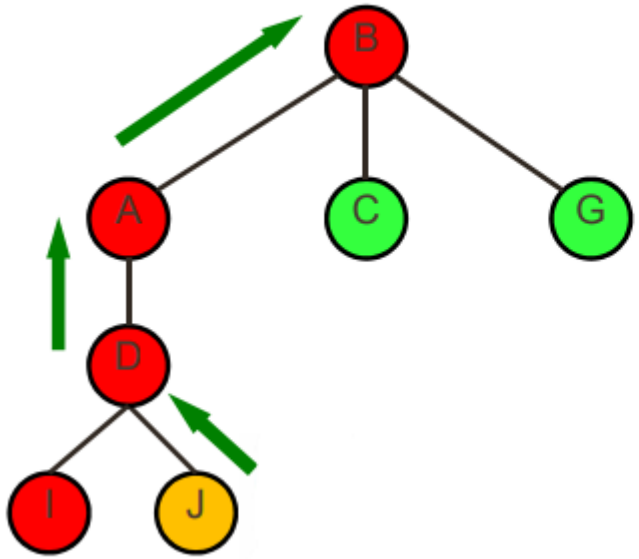
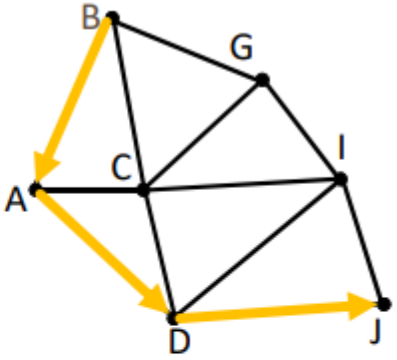
13  return FAILURE
```





Open (Q): {}

Closed: {B,A,C,G,D,I,J}

Final path solution: B→A→D→J

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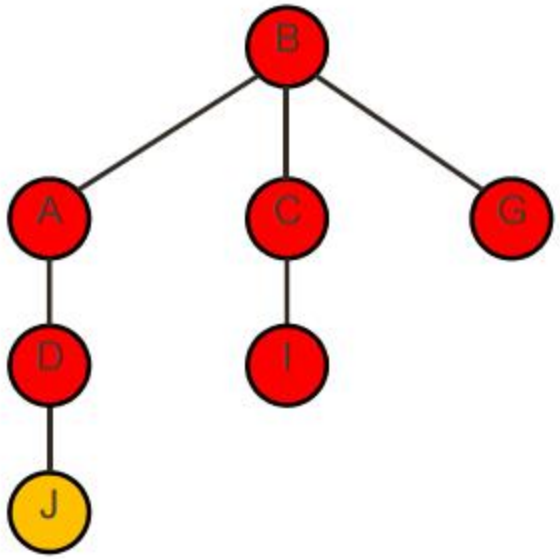
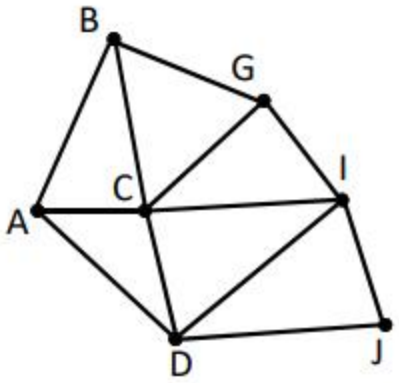
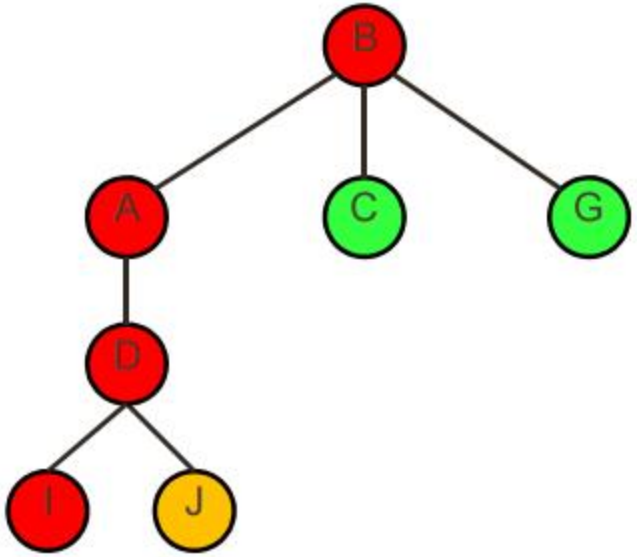
Uporedna analiza



BFS





DFS

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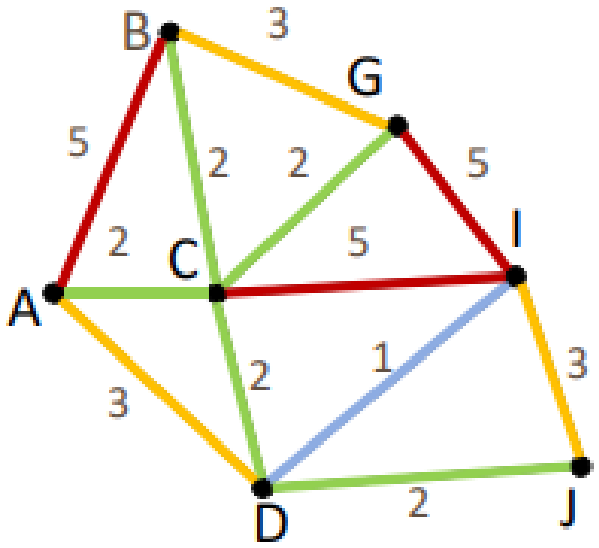
Akcije sa cenom

- Kako rešiti slučaj ako određena putanja ima veću cenu nego neka druga?
- Najpopularniji algoritmi Dijkstra's i A*



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Dijkstra's algoritam

- Objavljen od strane Edsger Dijsktra 1959.
- Otvara čvorove sa najmanjom cenom najbliže od korena grafa.
- Najkorišćeniji algoritam za rešavanje problema trgovačkog putnika.

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Dijkstra's algoritam

```
FORWARD SEARCH
1   Q.Insert(x_I) and mark x_I as visited
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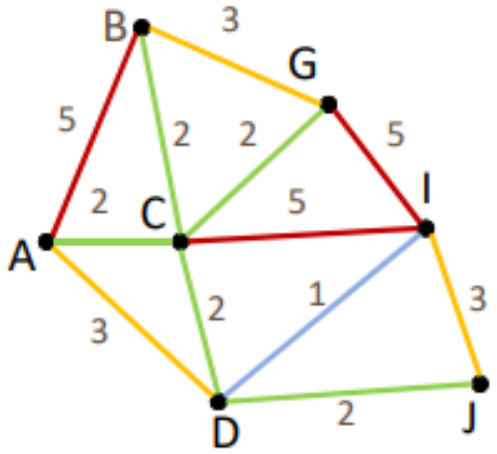
11          else
12              Resolve duplicate x'
13  return FAILURE
```

B(0)

Open (Q):	Closed:
{B(0)}	{B(0)}

Our Dijkstra queue will be ordered by cost to arrive:

- push (*Q.Insert*) by cost
- pop (*Q.GetFirst*) from the front, and add it to the closed list



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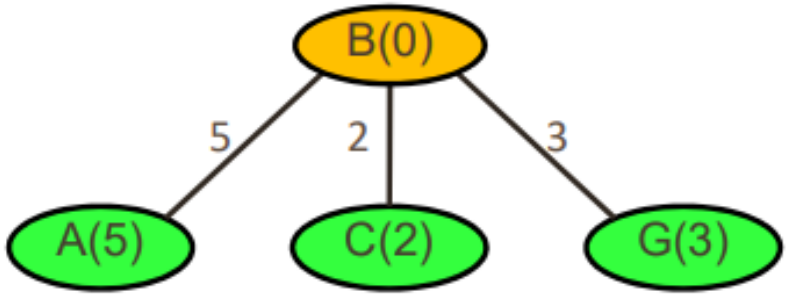
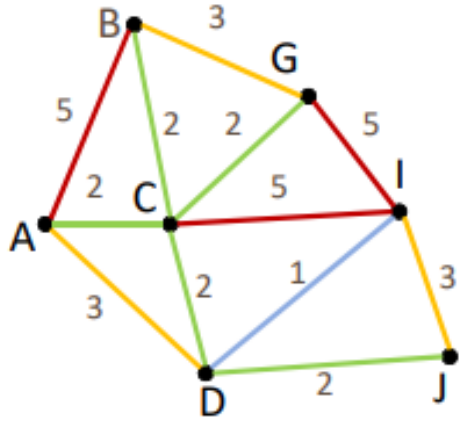
Dijkstra's algoritam

```
FORWARD_SEARCH
1   Q.Insert(x_I) and mark x_I as visited
2   while Q not empty do
3       x ← Q.GetFirst()
4       if x ∈ X_G
5           return SUCCESS
6       forall u ∈ U(x)
7           x' ← f(x, u)
8           if x' not visited
9               Mark x' as visited
10              Q.Insert(x')
11          else
12              Resolve duplicate x'
13  return FAILURE
```





Open (Q):
{ C (2),
G (3),
A (5) }

Closed:
{ B (0) }

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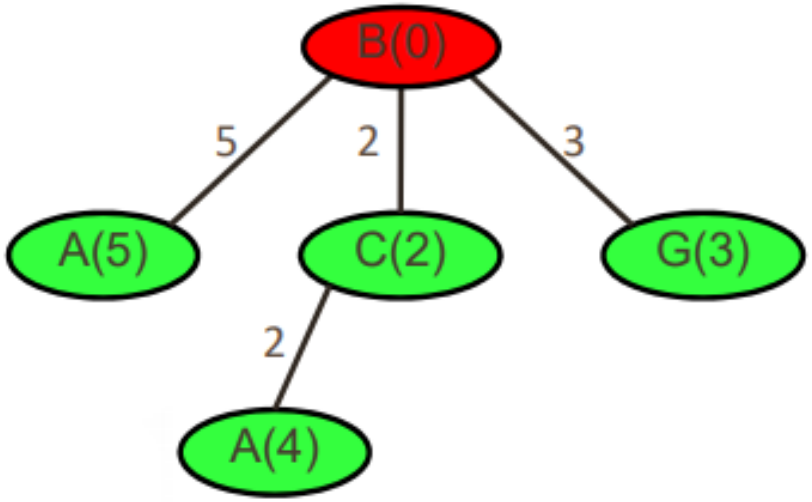
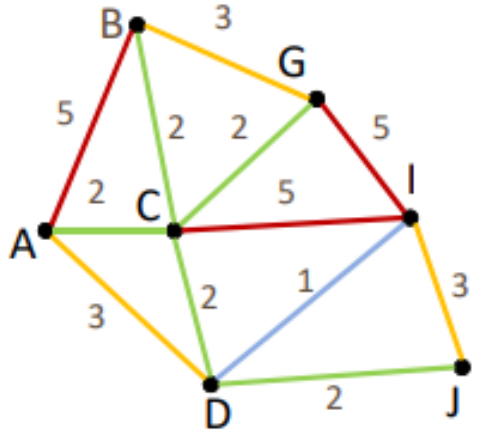
Dijkstra's algoritam

```
FORWARD_SEARCH
1   Q.Insert(x_I) and mark x_I as visited
2   while Q not empty do
3       x ← Q.GetFirst()
4       if x ∈ X_G
5           return SUCCESS
6       forall u ∈ U(x)
7           x' ← f(x, u)
8           if x' not visited
9               Mark x' as visited
10              Q.Insert(x')
11          else
12              Resolve duplicate x'
13  return FAILURE
```





Open (Q):
{ G (3),
A (4) }

Closed:
{ B (0),
C (2) }

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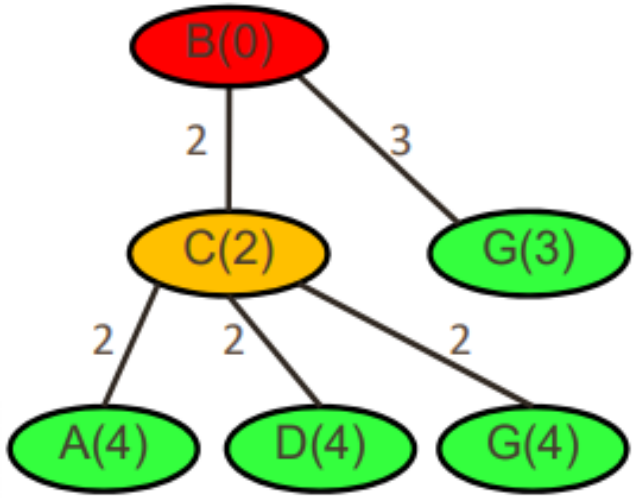
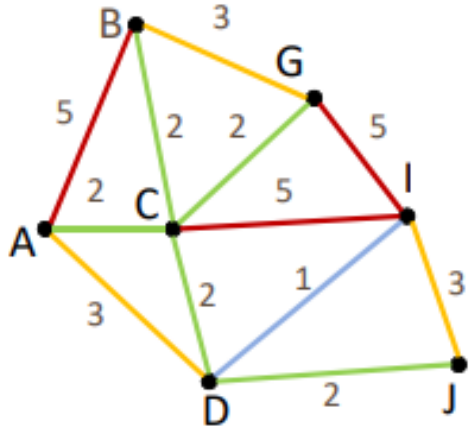
Dijkstra's algoritam

```
FORWARD_SEARCH
1   Q.Insert(x_I) and mark x_I as visited
2   while Q not empty do
3       x ← Q.GetFirst()
4       if x ∈ X_G
5           return SUCCESS
6       forall u ∈ U(x)
7           x' ← f(x, u)
8           if x' not visited
9               Mark x' as visited
10              Q.Insert(x')
11          else
12              Resolve duplicate x'
13  return FAILURE
```





Open (Q):
{ G (3),
A (4),
D (4) }

Closed:
{ B (0),
C (2) }

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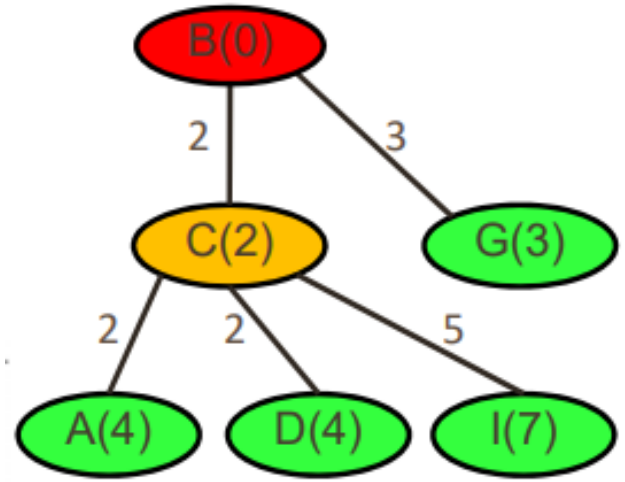
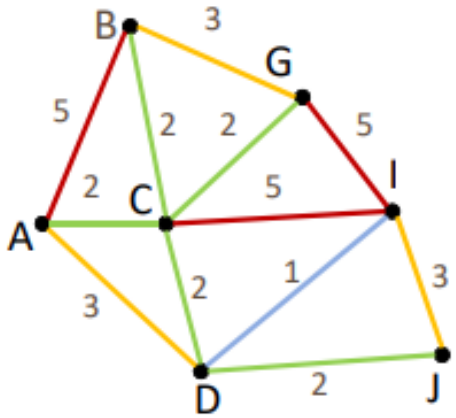
Dijkstra's algoritam

```
FORWARD_SEARCH
1   Q.Insert(x_I) and mark x_I as visited
2   while Q not empty do
3       x ← Q.GetFirst()
4       if x ∈ X_G
5           return SUCCESS
6       forall u ∈ U(x)
7           x' ← f(x, u)
8           if x' not visited
9               Mark x' as visited
10              Q.Insert(x')
11          else
12              Resolve duplicate x'
13  return FAILURE
```





Open (Q):
{ G (3),
A (4),
D (4),
I (7) }

Closed:
{ B (0),
C (2) }

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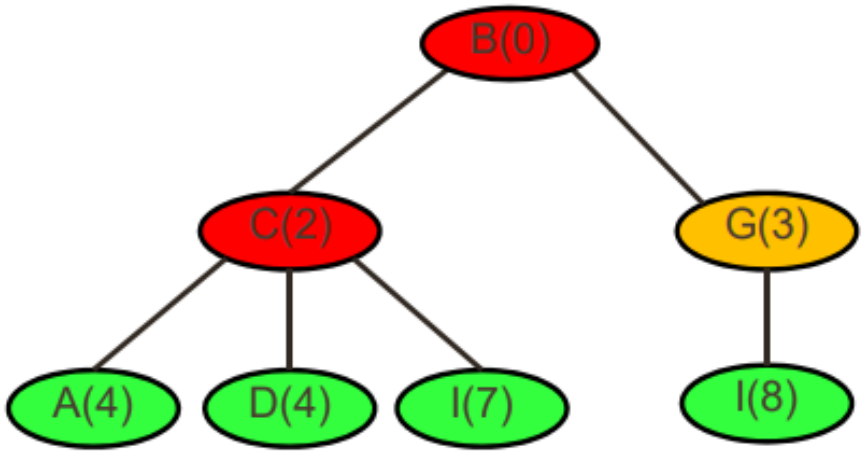
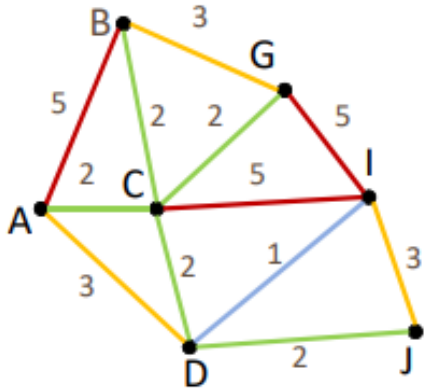
Dijkstra's algoritam





Open (Q):	Closed:
{ A (4),	{ B (0),
D (4),	C (2),
I (7) }	G (3) }

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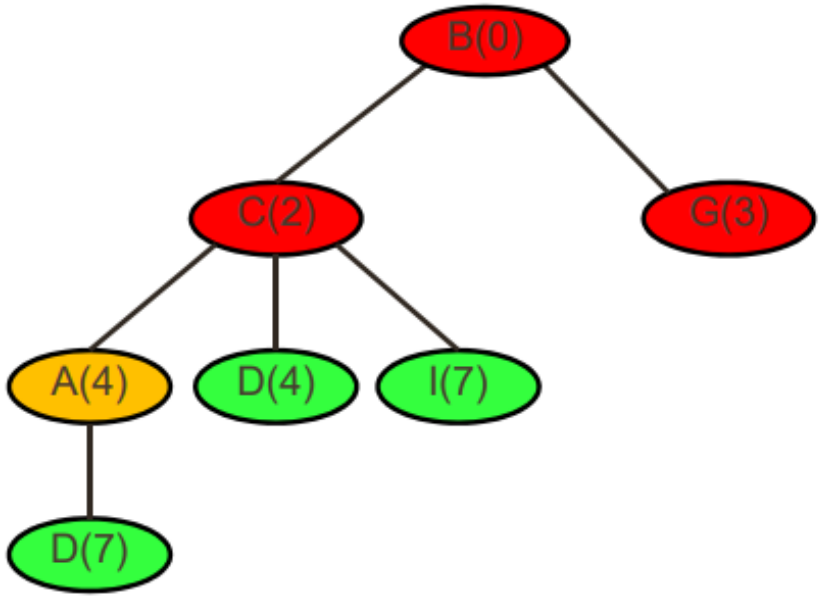
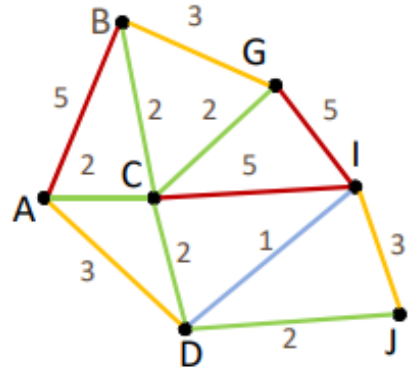
Dijkstra's algoritam





Open (Q):
{ D (4),
I (7) }

Closed:
{ B (0),
C (2),
G (3),
A (4) }

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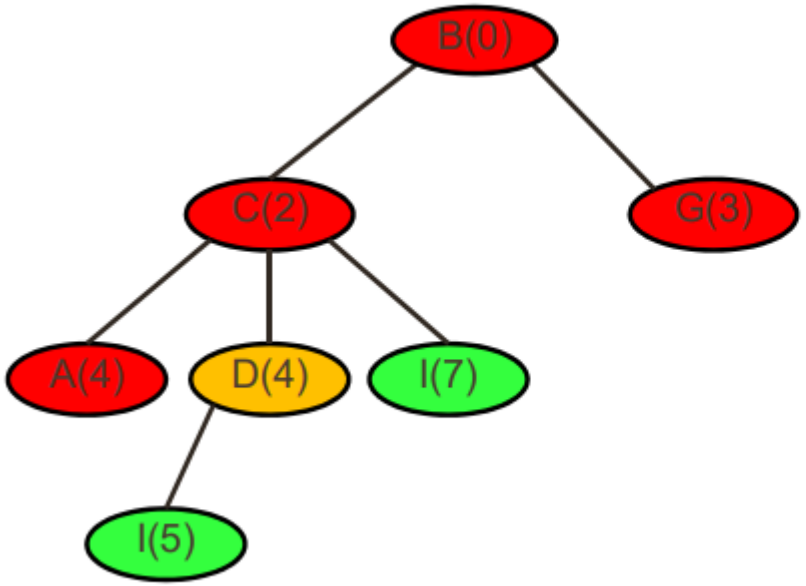
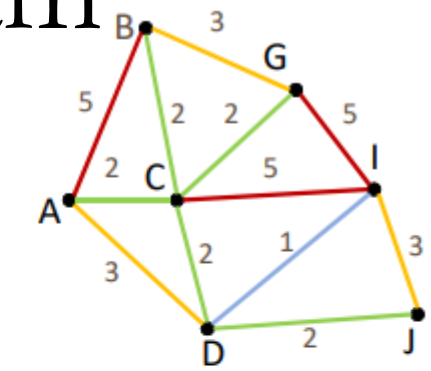
Dijkstra's algoritam





Open (Q):
{ I (5) }

Closed:
{ B (0),
C (2),
G (3),
A (4),
D (4) }

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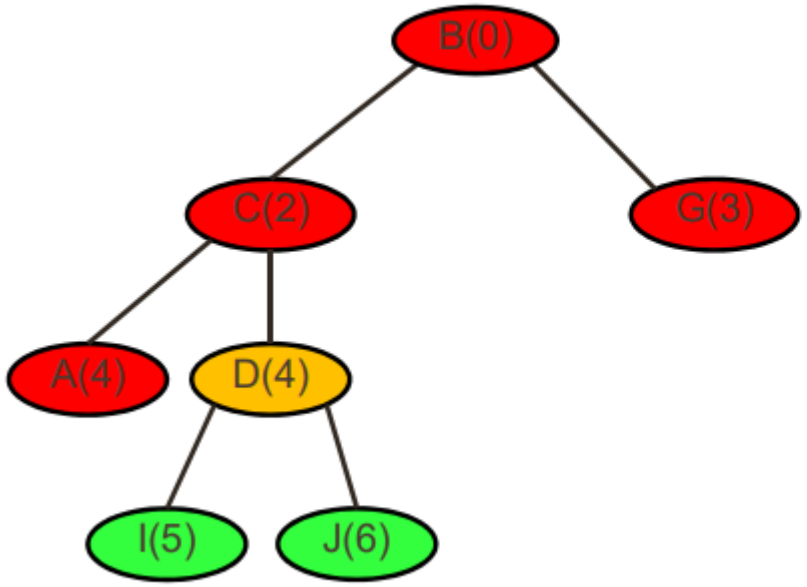
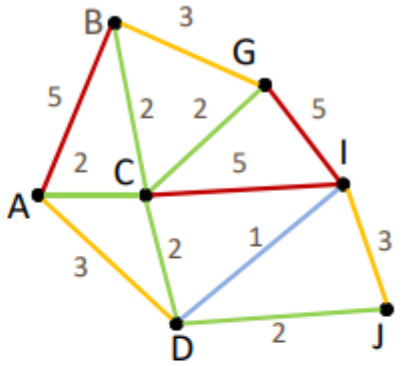
Dijkstra's algoritam





Open (Q):
{ I (5) ,
J (6) }

Closed:
{ B (0),
C (2),
G (3),
A (4),
D (4) }

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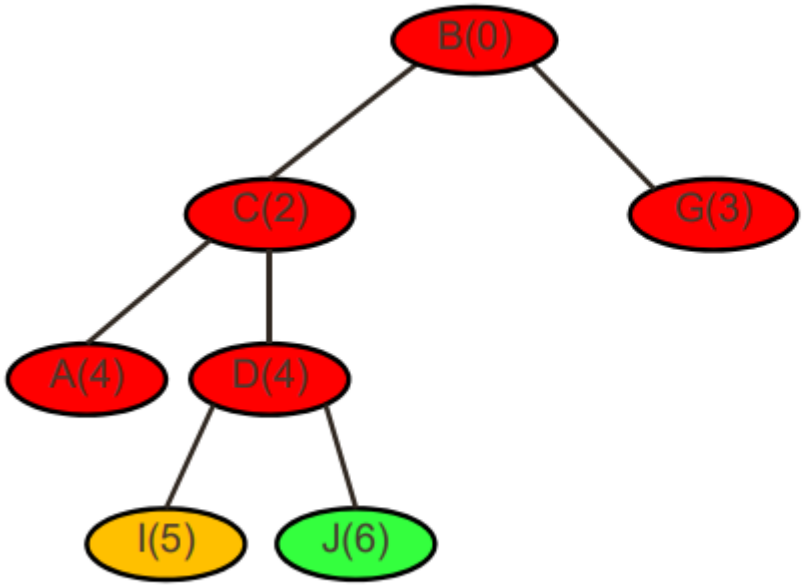
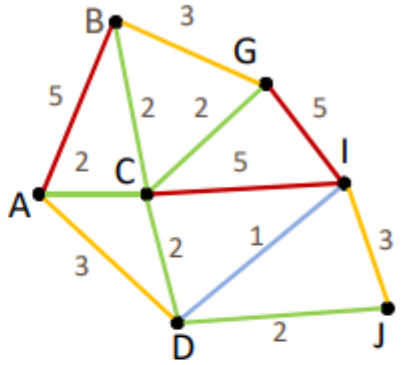
Dijkstra's algoritam





Open (Q):
{ J (6) }

Closed:
{ B (0),
C (2),
G (3),
A (4),
D (4),
I (5) }

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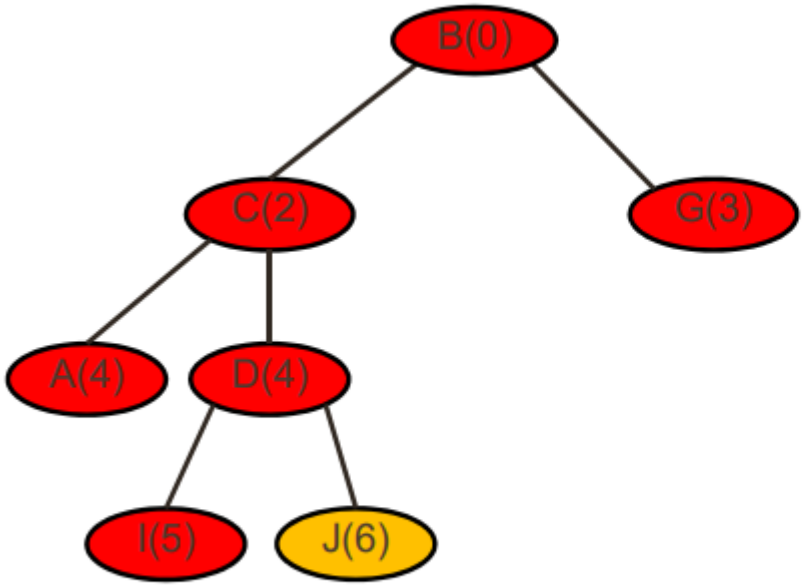
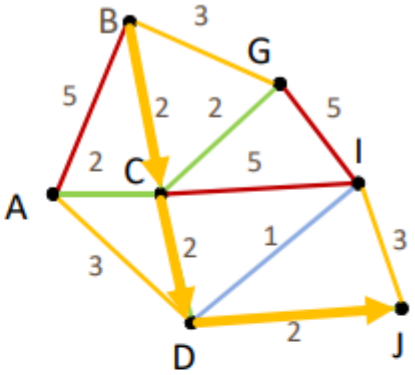
Dijkstra's algoritam





Open (Q):
{ }

Closed:
{ B (0),
C (2),
G (3),
A (4),
D (4),
I (5),
J (6) }

Final path solution: B→C→D→J
with path cost 6

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A* heuristička pretraga

- Heuristika:
  - Funkcija koja opisuje koliko smo daleko od cilja
  - Uvek se postavlja za određeni problem
  - Menhetn ili Euklidska distanca, …
- A* funkcija cene: $f(n) = g(n) + h(n)$

Cena puta

Cena heuristike

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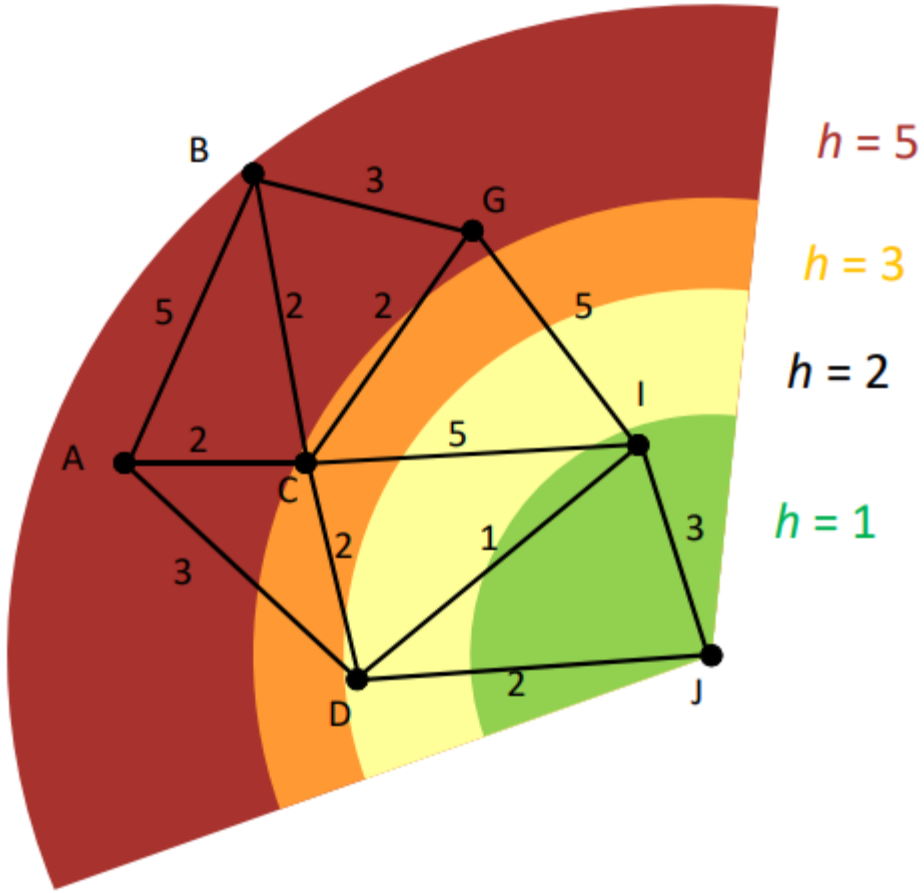
A* heuristička pretraga



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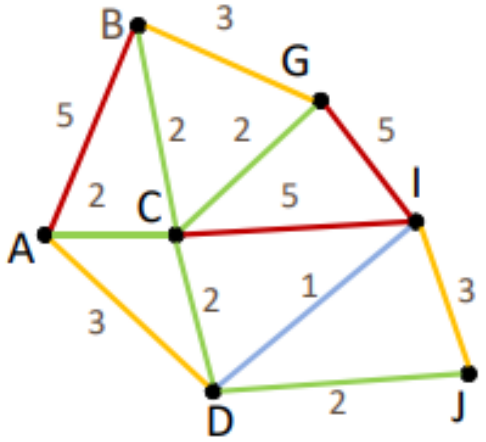
A* heuristička pretraga

Open (Q): {B(0)}

Closed: {B(0)}

B(0)



Our A* queue will be ordered by cost to arrive + heuristic:

- push (*Q.Insert*) by A* priority, *f(n)*
- pop (*Q.GetFirst*) from the front, and add it to the closed list

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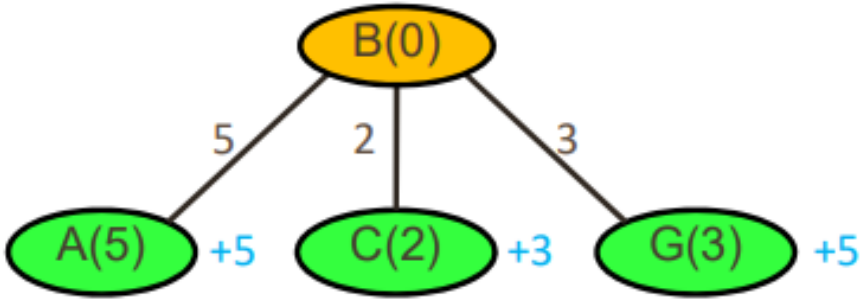
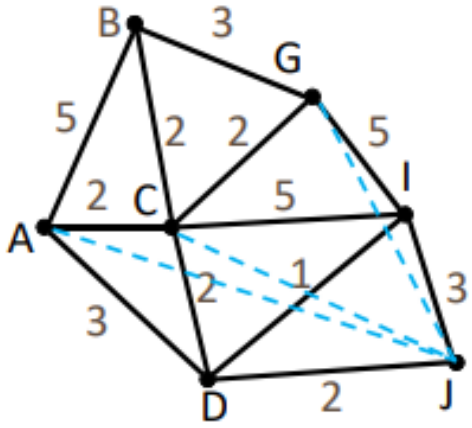
A* heuristička pretraga





Open (Q):
{ C (2+3),
G (3+5),
A (5+5) }

Closed:
{ B (0) }

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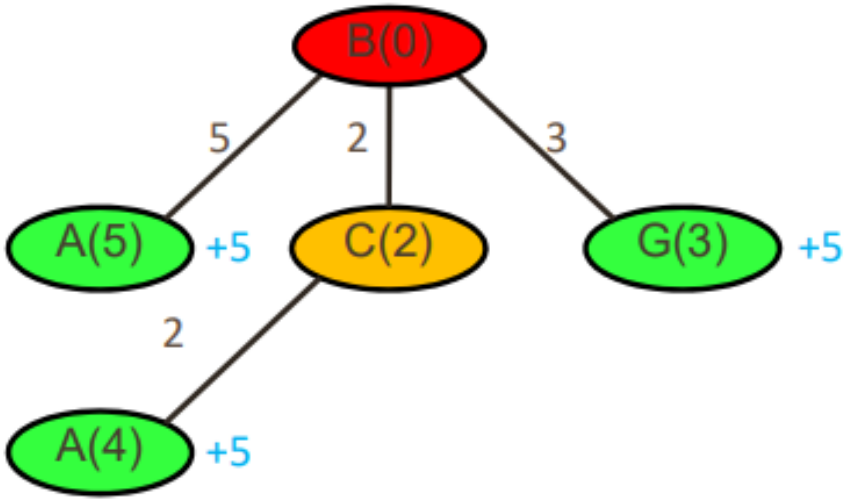
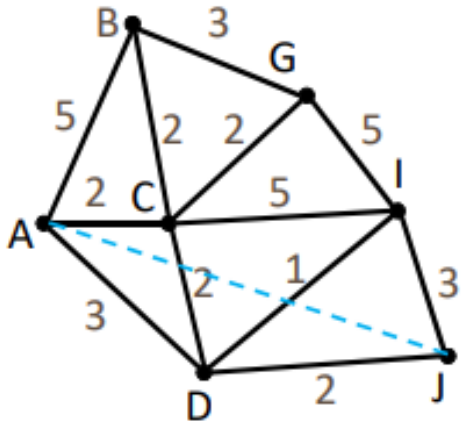
A* heuristička pretraga





Open (Q):
{ G (3+5),
A (4+5) }

Closed:
{ B (0) ,
C (2) }

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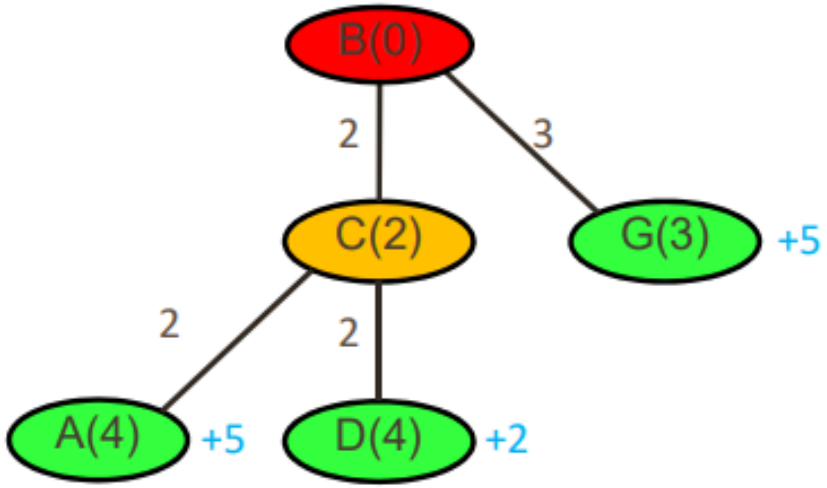
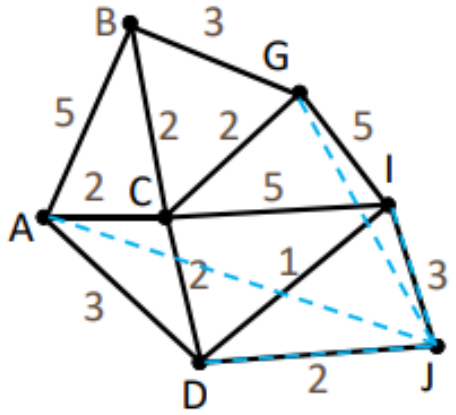
A* heuristička pretraga





Open (Q):
{ D (4+2),
G (3+5),
A (4+5) }

Closed:
{ B (0) ,
C (2) }

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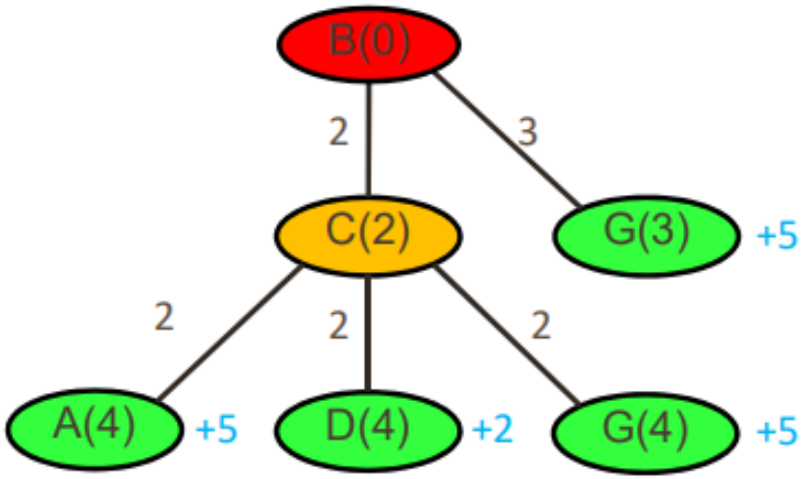
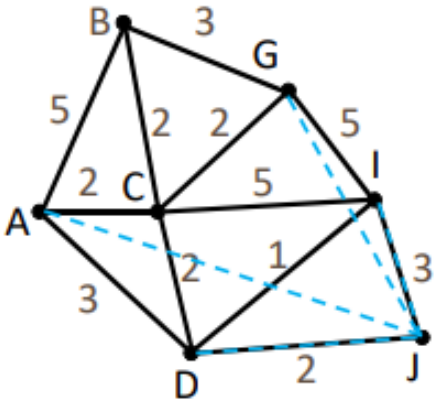
A* heuristička pretraga





Open (Q):
{ D (4+2),
G (3+5),
A (4+5) }

Closed:
{ B (0) ,
C (2) }

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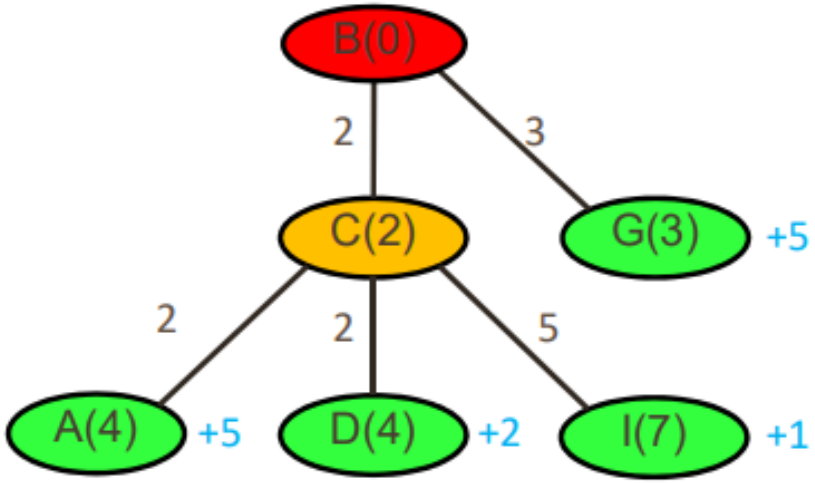
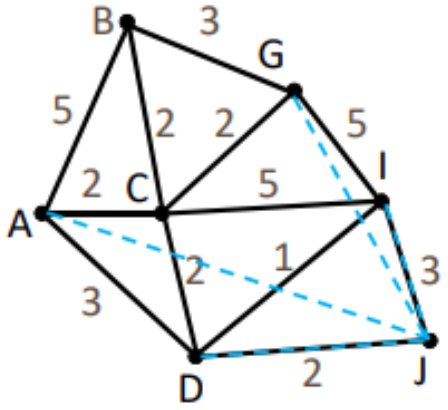
A* heuristička pretraga





Open (Q):
{ D (4+2),
I (7+1),
G (3+5),
A (4+5) }

Closed:
{ B (0) ,
C (2) }

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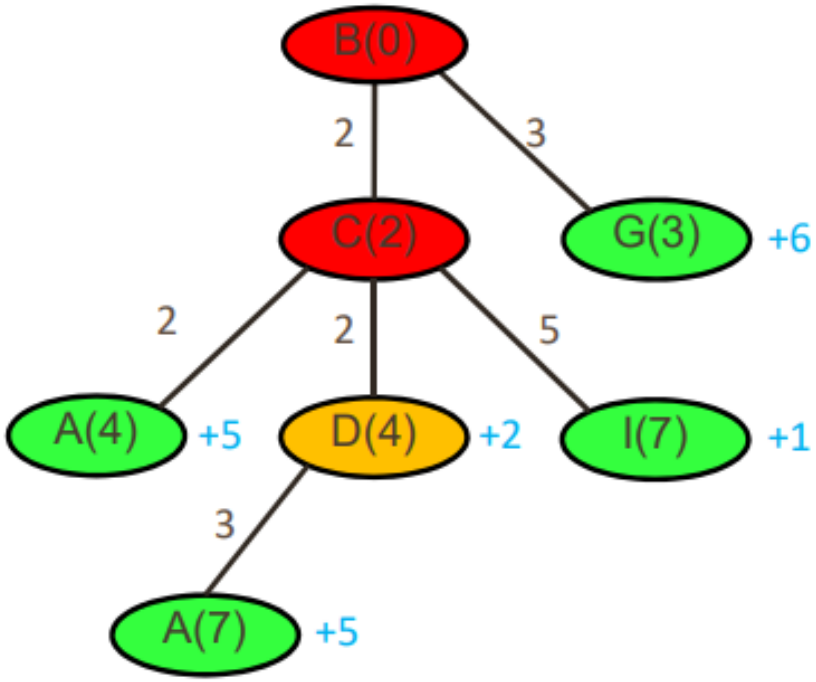
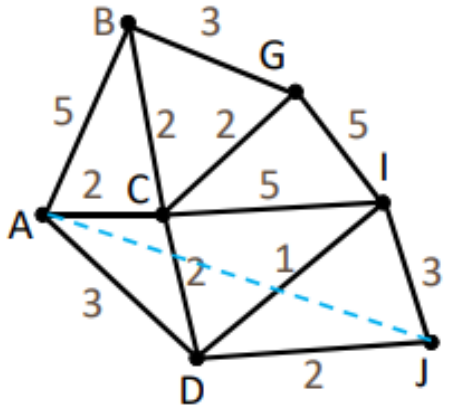
A* heuristička pretraga





Open (Q):
{ I (5+1),
G (3+5),
A (4+5) }

Closed:
{ B (0) ,
C (2),
D (4) }

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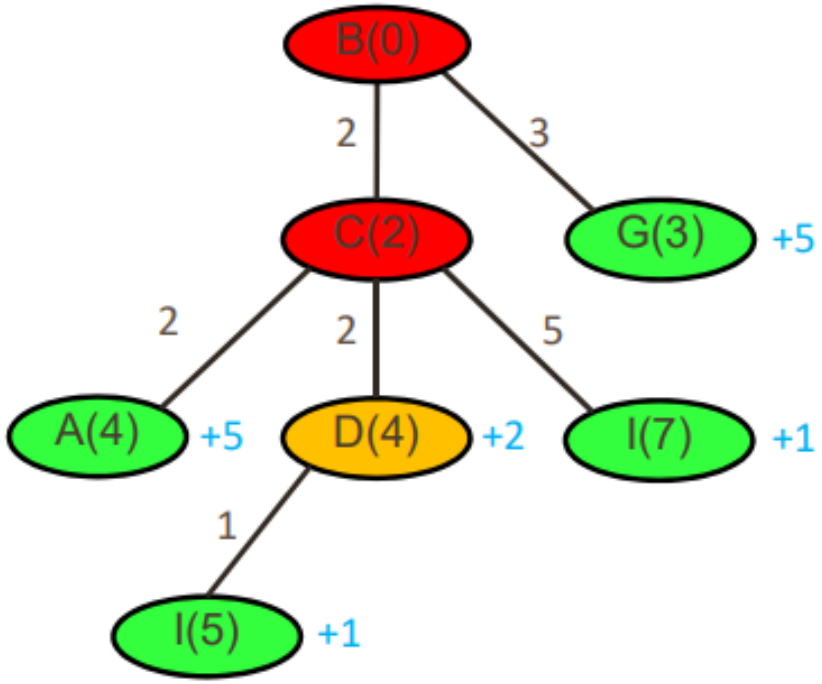
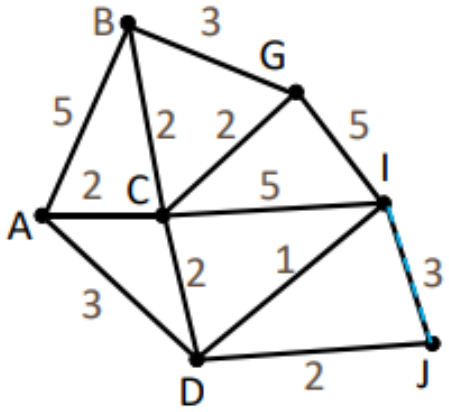
A* heuristička pretraga





Open (Q):
{ I (5+1),
G (3+5),
A (4+5) }

Closed:
{ B (0) ,
C (2),
D (4) }

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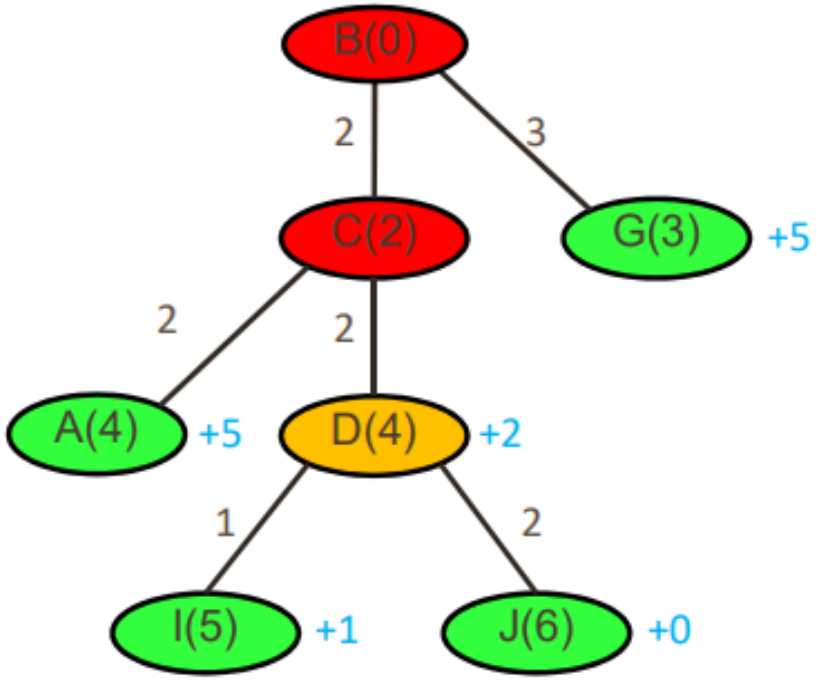
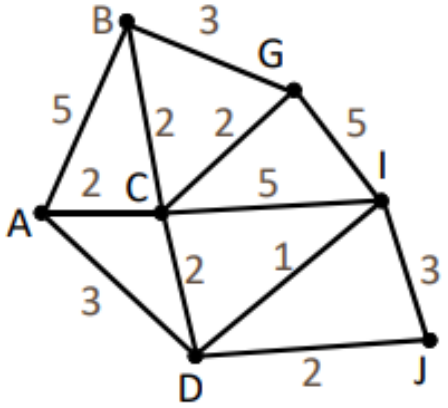
A* heuristička pretraga





Open (Q):
{ J (6+0),
I (5+1),
G (3+5),
A (4+5) }

Closed:
{ B (0) ,
C (2),
D (4) }

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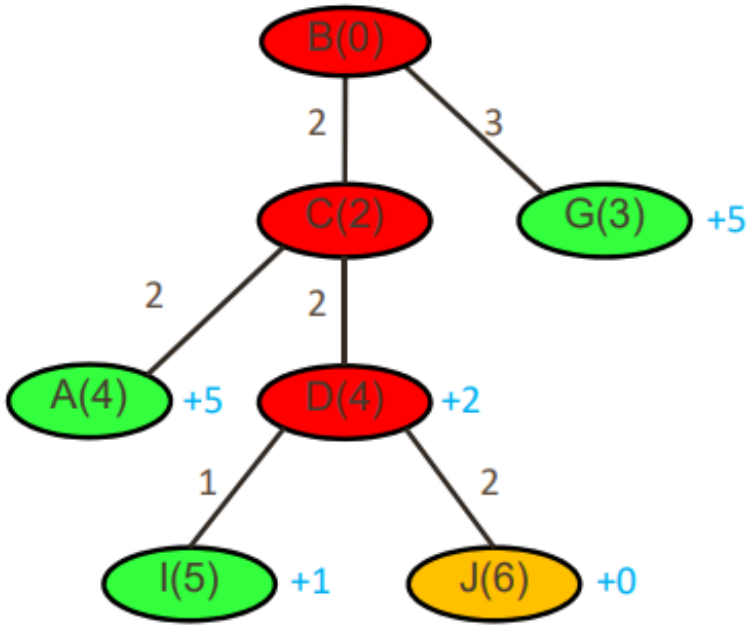
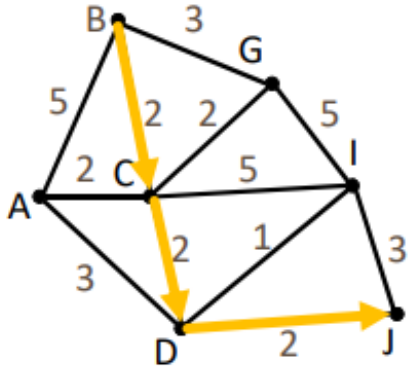
A* heuristička pretraga





Open (Q):
{ I (5+1),
G (3+5),
A (4+5) }

Closed:
{ B (0) ,
C (2),
D (4),
J (6) }

Final path solution: B→C→D→J
with path cost 6

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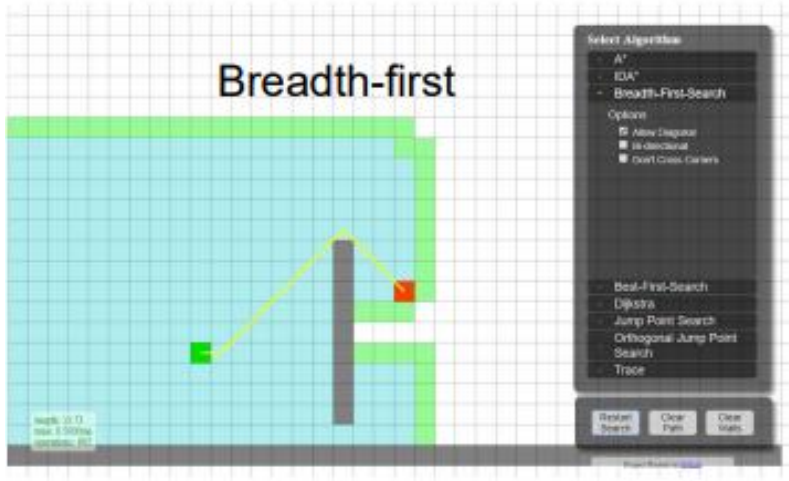
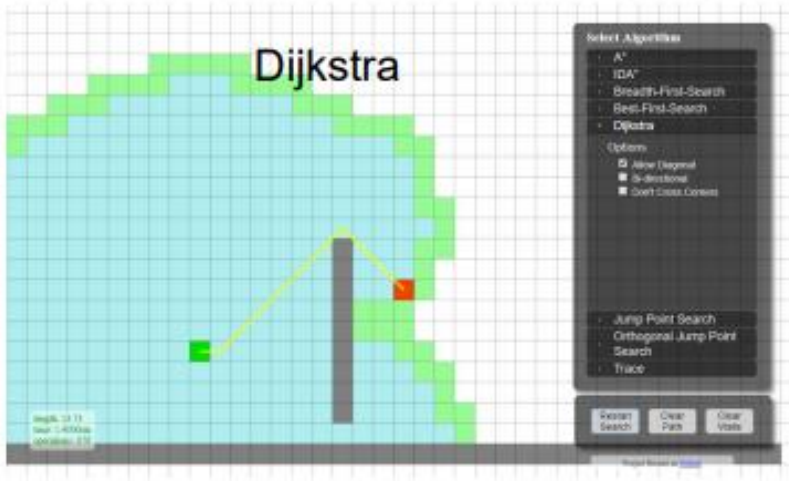
https://qiao.github.io/PathFinding.js/visual/





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Thanks!

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